3D BEM Formulations for Eddy Current Problems with Multiply Connected Domains and Circuit Coupling

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Quasi-static linear problems can be solved efficiently with Boundary Element Method (BEM). This method is based on surface integral equations dealing with equivalent magnetic and electric surface current densities. Many works have shown the potentiality of BEM especially for the modeling of non-destructive testing devices. In this paper, after selecting formulations enabling the modeling of multiply-connected regions, an original coupling is proposed in order to take into account external electric circuit in the problem.

***Index Terms*—Boundary Element Method, eddy currents, quasi-static, coupling circuits.**

I. Introduction

THIS study is motivated by the need of efficient low-frequency electromagnetic formulations associated to non-simply connected domains and which can be coupled with circuit equations. For this kind of problems, methods based on coupling between BEM and Finite Element Method (FEM) have already shown very good results. However, the accuracy of these methods is strongly influenced by the volume mesh of the conductive region which has to be adapted to the skin-depth in the FEM context. The pure BEM method does not suffer from this disadvantage because it is only based on surface discretizations and is more insensitive to frequency increase keeping the same mesh.

Many BEM formulations have already been proposed to solve Eddy Current (EC) problems and two options are possible. The first one consists in solving the full Maxwell equations including capacitive effects and to use an Helmholtz-Hodge decomposition to remove low-frequency numerical noise like proposed in [1]. The second option is to solve directly Maxwell equations under the quasi-static assumption (i.e the EC problem) like proposed in [2],[3] or [4]. In this work, the preference is given to the last approach.

In classical EC problems (like eddy current testing problems for instance), the active region can be multiply connected, meaning that it can have through-holes. It is fundamental to develop formulations which can solve accurately such a problem. An another key point is the ability of the formulation to be coupled with external electric circuits which create the external inductor field. This is what we aim to do in this paper. After selecting efficient BEM formulations for EC problems, an original circuit coupling is proposed. All the formulations are validated on numerous examples by a comparison with FEM.

II. Surface Integral Formulation

Let us consider a linear, isotropic and homogeneous conducting material region Ω characterized by the conductivity σ_1 and the permeability μ_1 and embedded in the free space Ω_0. Γ is the border of the region Ω with the outward unit normal $\mathbf{n}$. Surface integral equations of $\mathbf{E}$ and $\mathbf{H}$ at an arbitrary target point P in Ω_0 are :

$$h\mathbf{H} = \mathbf{H}_{ex} + \int_\Gamma \left((\mathbf{n} \cdot \mathbf{H}_0) \nabla G_0 + \mathbf{J}_s \times \nabla G_0 \right) d\Gamma \tag{1}$$

$$\begin{aligned} h\mathbf{E} = \mathbf{E}_{ex} &+ \int_\Gamma \left((\mathbf{n} \cdot \mathbf{E}_0) \nabla G_0 - \mathbf{M}_s \times \nabla G_0 \right) d\Gamma \\ &- j\omega\mu_0 \int_\Gamma \mathbf{J}_s G_0 d\Gamma \end{aligned} \tag{2}$$

and for a target point in Ω are :

$$\begin{aligned} h\mathbf{H} = &- \int_\Gamma \left(\frac{\mu_0}{\mu_1} (\mathbf{n} \cdot \mathbf{H}_0) \nabla G_1 - \sigma_1 \mathbf{M}_s G_1 \right) d\Gamma \\ &- \int_\Gamma \mathbf{J}_s \times \nabla G_1 d\Gamma \end{aligned} \tag{3}$$

$$h\mathbf{E} = \int_\Gamma (\mathbf{M}_s \times \nabla G_1 + j\omega\mu_1 \mathbf{J}_s G_0) d\Gamma \tag{4}$$

where $h = 0.5$ if P is a regular point of the border Γ, $h = 1$ if P belongs to Ω or Ω_0. The Green kernels expressions are $G_0 = \frac{1}{r}$ and $G_1 = \frac{1}{4\pi r} e^{-(1+j)kr}$ with $k = \sqrt{\frac{\omega\mu_1\sigma_1}{2}}$, $\mathbf{H}_0$ and $\mathbf{E}_0$ are the magnetic and electric fields in the air region respectively, $\mathbf{H}_{ex}$ is the inductor magnetic field and $\mathbf{E}_{ex}$ is the inductor electric field. $\mathbf{J}_s$ and $\mathbf{M}_s$ are equivalent fictitious magnetic and electrical surface currents such as :

$$\mathbf{J}_s = \mathbf{n} \times \mathbf{H} \tag{5}$$

$$\mathbf{M}_s = -\mathbf{n} \times \mathbf{E} \tag{6}$$

From the four equations above and by considering their normal and tangential projections on Γ, we get eight integral boundary equations. By combining these integral equations like presented in Table I, one obtains two formulations whom one calls EC1 and EC2. Let us notice that EC1 has already been presented in [3] and EC2 in [1], [4]. Both formulations can treat multiply connected problems and can be coupled with external electric circuits.

TABLE I
FORMULATIONS

Label	Formulations	Unknown variables
EC1	$\mathbf{n}\times[(1)+(3)]$, $\mathbf{n}\times[(2)+(4)]$, $\mathbf{n}\cdot[(2)+(4)]$	$\mathbf{J}_s$, $\mathbf{M}_s$, $\mathbf{n}\cdot\mathbf{E}_0$
EC2	$\mathbf{n}\times[(1)-(3)]$, $\mathbf{n}\times[(2)-(4)]$, $\mathbf{n}\cdot[(2)]$	$\mathbf{J}_s$, $\mathbf{M}_s$, $\mathbf{n}\cdot\mathbf{E}_0$

TABLE II
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Frequency (Hz)		10	100	1000	10000
Error (%)	EC1	0.99	0.72	0.98	1.75
	EC2	0.33	0.16	0.93	1.95

III. CIRCUIT COUPLING

In the context of circuit coupling, the conducting region Ω is excited by external electrical circuits Ω_k connected to voltage sources U_k. The equation governing the k^{th} electric circuit is presented in [5]:

$$U_k = I_k R_k + j\omega \int\limits_{\Omega_k} \mathbf{j}_{0k}.\mathbf{A} d\Omega. \tag{7}$$

where R_k is the ohmic resistance of Ω_k, I_k is the total current flowing in the circuit, $\mathbf{j_{0k}}$ is the normalized (1A) current density associated to Ω_k and $\mathbf{A}$ is a total magnetic vector potential.

Let us consider equation (2). By setting the coefficient h to 1 and thanks to the relation $\mathbf{E} = -j\omega\mathbf{A}$, we get a vector relation linking the unknowns to $\mathbf{A}$ in the air region and $\mathbf{A}_{ex}$. $\mathbf{A}_{ex}$ is linked to I_k by the Biot-Savart law which can be integrated numerically over Ω_k. In a symmetrical way, the vector potential in (7) can be expressed with problem unknowns. This step and all the matrix coefficients will be explained in the full paper but an overview of the matrix system is presented in Fig. 1.

Formulations EC1 or EC2 and their coupling with electric circuit are solved at once. Unknown variables in our system consist of $\mathbf{M}_s$, $\mathbf{J}_s$, $\mathbf{n}\cdot\mathbf{E}_0$ and $\{I_k\}$.

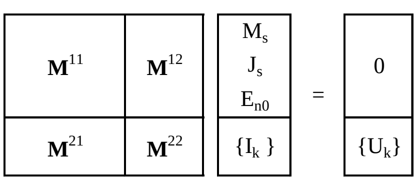

Fig. 1. Representation of the matrix system for the coupling.

IV. NUMERICAL RESULTS

A first simple test with a non-magnetic and conducting axisymmetric torus $\sigma = 5.5E7(S/m)$ excited by a circular coil with current of $1000(A)$ is proposed. Obtained results are presented in Table II and Fig. 2 with the reference computed by 2D axisymmetric FEM. The geometry is meshed with about 2.300 quadrangle elements. It can be seen that results remains quite good even if the skin depth highly decrease keeping the mesh unchanged. This is one of the main advantage of BEM method compared to FEM where the mesh in the volume has to be adapted.

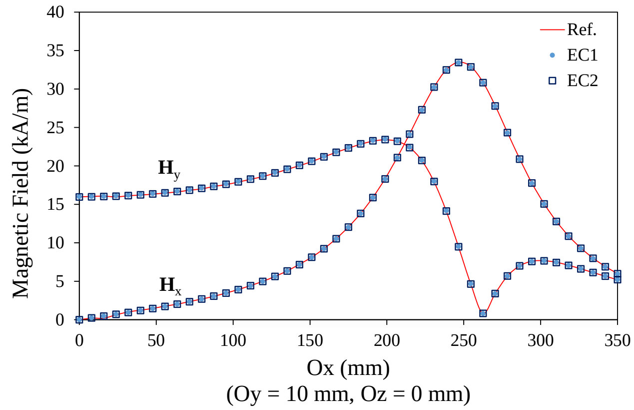

Fig. 2. Comparisons of magnetic field computed on a line located in the air region.

The circuit coupling is now tested. A square section ring excited by an electric circuit described in Table III is considered. The reference is calculated by FEM with 2D axisymmetric model and a circuit coupling. The total currents flowing in the electric circuit are compared in Table IV. The difference is very low even if the frequency is high validating the coupling and demonstrating its performance.

TABLE III
PARAMETERS DESCRIPTION

Square section ring		Electrical circuit	
Conductivity	3.526E7 (S/m)	U_k	10 (V)
Relative permeability	1	Height/width	5 (mm)
Height	60 (mm)	Number of turns	100
External radius	25 (mm)	External radius	31 (mm)
Internal radius	10 (mm)	Internal radius	26 (mm)

TABLE IV
CURRENT IN THE ELECTRICAL CIRCUIT

Frequency (Hz)	10	1000	10000
Current (A)	2.4589	1.7536	0.3409
Error (%)	0.0028	0.1570	1.7603

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