Parallel TP-EEC Method based on Polyphase Time-periodic Condition for Magnetic Field Analysis of Induction Motors

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This paper develops the parallel time-periodic explicit error correction (TP-EEC) method considering a polyphase time-periodic condition for magnetic field analyses of cage induction motors. In addition, the convergence behavior of the transient solution to its steady-state is improved by appropriately setting the initial values of the parallel TP-EEC method in each process by nonlinear time-harmonic analyses. Numerical results of a cage induction motor are presented to demonstrate the effectiveness of the proposed method.

***Index Terms*— Finite-element method, induction motor, parallel computing, polyphase time-periodic condition.**

I. Introduction

Recently, a parallel computing based on a finite-element method (FEM) is used in designing electric machines. A domain decomposition method (DDM) [1], [2] is a typical parallel computing technique suitable for a large-scale problem. On the other hand, the magnetic field analyses of rotating machines fed by pulse-width modulation (PWM) inverters need a sufficiently-small time step size to consider carrier harmonics. In this case, the computation time required for obtaining steady-state solutions has a tendency to become huge even if the problem is 2-dimensional (2-D). Because the problem size is small, the use of the DDM is of limited effectiveness. To overcome the difficulty, parallel-in-time (PinT) integration methods have been reported [3]-[7].

This paper focuses on the steady-state analysis of induction machines by the PinT integration method. Because of the slip, the period in the rotor is generally much longer than that in the stator. Therefore, it is not straightforward to apply ordinary PinT integration methods to steady-state analyses of induction machines because of the different time-periodicity in the rotor and stator. In [4], with the help of a polyphase time-periodic condition, the time domain parallel FEM (TDPFEM) specialized for cage induction motors was proposed. In this paper, we introduce the polyphase time-periodic condition into the formulation of the parallel time-periodic explicit-error-correction (TP-EEC) method [6], which is one of the PinT integration methods. Furthermore, to improve the convergence behavior of numerical transients to the steady-state, the initial values of the parallel TP-EEC method are appropriately determined in each process by a nonlinear time-harmonic eddy-current analysis [8]. The parallel performance of the TDPFEM and the parallel TP-EEC method is clarified in a magnetic field analysis of an induction motor fed by a PWM inverter.

II. Method of Analysis

A. Polyphase Time-periodic Condition for Induction Motor

A polyphase time-periodic condition is satisfied in the rotor region of an induction machine when the slip s is given by [4]

$$s = (rP - qN_b)/(mN_b), \tag{1}$$

where m, r, q are integers which satisfy $1 \le m$, $0 \le r < mN_b/P$ and $0 \le q < P$, N_b is the number of rotor bars per P poles. The slip in (1) means that the rotor bar with bar number B ($0 \le B < N_b$) moves to the position of the rotor bar with bar number mod($B-r$, N_b) at $t = 0$ in $m/2$ periods.

For simplicity, we consider an example shown in Fig. 1(a) in which $P = 2$ and $N_b = 9$. When m=2, r=1 and q=0, we have s=1/9 and the position of the rotor bar 0 at t=T corresponds to that of the rotor bar 8 at t=0 as shown in Fig. 1(b), where T is the period. In this case, the following nine-phase time periodic condition should be satisfied in the rotor region:

$$\begin{Bmatrix} \boldsymbol{x}_0^{R0} \\ \boldsymbol{x}_0^{R1} \\ \vdots \\ \boldsymbol{x}_0^{R7} \\ \boldsymbol{x}_0^{R8} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{x}_n^{R1} \\ \boldsymbol{x}_n^{R2} \\ \vdots \\ \boldsymbol{x}_n^{R8} \\ \boldsymbol{x}_n^{R0} \end{Bmatrix} = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ \vdots & 0 & I & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & & & \ddots & I \\ I & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_n^{R0} \\ \boldsymbol{x}_n^{R1} \\ \vdots \\ \boldsymbol{x}_n^{R7} \\ \boldsymbol{x}_n^{R8} \end{Bmatrix} = H_{N_b} \begin{Bmatrix} \boldsymbol{x}_n^{R0} \\ \boldsymbol{x}_n^{R1} \\ \vdots \\ \boldsymbol{x}_n^{R7} \\ \boldsymbol{x}_n^{R8} \end{Bmatrix}, \tag{2}$$

where H_{Nb} is the conversion matrix representing the polyphase time-periodic condition, $\boldsymbol{x}$ is the unknown vector associated with rotor bars, the subscript and the superscript indicate the time step and the rotor bar number, respectively, and n is the number of time steps per period.

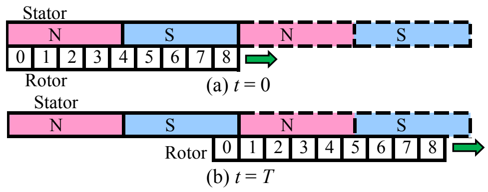

Fig. 1. Example of polyphase time-periodic condition in induction motors.

B. Parallel TP-EEC Method Considering Polyphase Time-periodic Condition

A*-*φ formulation is used for a finite-element analysis in a quasi-static field, where $\boldsymbol{A}$ is the magnetic vector potential and φ is the electric scalar potential. Here, the number of time steps per (half) period n_S can be divided by the number of processes n_P, and l=n_S/n_P is the number of time steps assigned to each process. In the parallel TP-EEC method, first, the following l nonlinear equations are solved at process k ($0 \le k < n_P$)

$$\begin{cases} \boldsymbol{S}(\boldsymbol{x}_{lk+1}) + \dfrac{1}{\Delta t} \boldsymbol{C} \boldsymbol{x}_{lk+1} = \dfrac{1}{\Delta t} \boldsymbol{C} \boldsymbol{x}_0^k + \boldsymbol{f}_{lk+1} \\ \qquad \vdots \\ \boldsymbol{S}(\boldsymbol{x}_{l(k+1)}) + \dfrac{1}{\Delta t} \boldsymbol{C} \boldsymbol{x}_{l(k+1)} = \dfrac{1}{\Delta t} \boldsymbol{C} \boldsymbol{x}_{l(k+1)-1} + \boldsymbol{f}_{l(k+1)} \end{cases}, \qquad (3)$$

where $\boldsymbol{x}$ is the unknown vector, $\boldsymbol{f}$ is the right-hand-side vector, $\boldsymbol{C}$ is a constant matrix, $\boldsymbol{x}_0^k$ denotes the initial value in process k, and the subscript indicates the time step. $\boldsymbol{S}(\boldsymbol{x})$ is nonlinear with respect to $\boldsymbol{x}$ because of the nonlinear magnetic properties.

In the steady state of cage induction machines, the solutions in the rotor region must satisfy

$$\begin{pmatrix} \boldsymbol{x}_0^0 \\ \boldsymbol{x}_0^1 \\ \vdots \\ \boldsymbol{x}_0^{n_\mathrm{P}-1} \end{pmatrix} = \begin{pmatrix} \pm H_{N_\mathrm{b}} \boldsymbol{x}_{n_\mathrm{S}} \\ \boldsymbol{x}_l \\ \vdots \\ \boldsymbol{x}_{l(n_\mathrm{P}-1)} \end{pmatrix} = \begin{pmatrix} O & \cdots & O & \pm H_{N_\mathrm{b}} \\ I & \ddots & & O \\ & \ddots & \ddots & \vdots \\ O & & I & O \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_l \\ \boldsymbol{x}_{2l} \\ \vdots \\ \pm \boldsymbol{x}_{n_\mathrm{S}} \end{pmatrix} \qquad (4)$$

by considering the polyphase time-periodic condition. In the parallel TP-EEC method [6], the initial values that satisfy the condition (4) is found by solving

$$\begin{bmatrix} \tilde{\boldsymbol{C}} + \sum_{i=1}^{l} \boldsymbol{S}_i & O & \cdots & O & \mp \tilde{\boldsymbol{C}} H_{N_\mathrm{b}} \\ -\tilde{\boldsymbol{C}} & \ddots & \ddots & & O \\ O & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & O \\ O & \cdots & O & -\tilde{\boldsymbol{C}} & \tilde{\boldsymbol{C}} + \sum_{i=l(n_\mathrm{P}-1)+1}^{n_\mathrm{S}} \boldsymbol{S}_i \end{bmatrix} \begin{pmatrix} \boldsymbol{p}^0 \\ \boldsymbol{p}^1 \\ \vdots \\ \boldsymbol{p}^{n_\mathrm{P}-1} \end{pmatrix} = \begin{pmatrix} \tilde{\boldsymbol{C}}\left(\pm H_{N_\mathrm{b}} \boldsymbol{x}_{n_\mathrm{S}} - \boldsymbol{x}_0^0\right) \\ \tilde{\boldsymbol{C}}\left(\boldsymbol{x}_l - \boldsymbol{x}_0^1\right) \\ \vdots \\ \tilde{\boldsymbol{C}}\left(\boldsymbol{x}_{l(n_\mathrm{P}-1)} - \boldsymbol{x}_0^{n_\mathrm{P}-1}\right) \end{pmatrix}, \qquad (5)$$

where $\boldsymbol{S}_i = \partial \boldsymbol{S}(\boldsymbol{x}_i)/\partial \boldsymbol{x}_i$, $\tilde{\boldsymbol{C}} = \boldsymbol{C}/\Delta t$, $\boldsymbol{p}^k$ is the correction vector in process k. Because (4) can be regarded as a kind of n_P phase time-periodic conditions, the system of auxiliary equations for the parallel TP-EEC method is finally given by (5). After solving (5), the initial values for each transient calculation are updated by $\boldsymbol{x}_0^{k+1} \leftarrow \boldsymbol{x}_{l(k+1)} + \boldsymbol{p}^k$ and the transient calculation in (3) is repeated until the steady-state solutions are obtained.

To improve the convergence behavior of numerical transients to the steady-state in ordinary magnetic field analyses of induction motors, it is important to appropriately select the initial values at $t = 0$ in (3) [9]. In this paper, the initial values of independent transient calculations in (3) are obtained by the nonlinear time-harmonic eddy-current analysis [8].

III. Numerical Results

To investigate the effectiveness of the proposed method, 2-D magnetic field analysis of the cage induction motor in [4] are performed at s=1/17. Because P=2 and N_b = 17 in the motor, the set of (m, r, q) in (1) which satisfies s=1/17 is either (2, 1, 0) or (1, 9, 1). Fig. 2 shows the time variation of the eddy-current loss in the secondary conductor under sinusoidal excitation. By starting the transient calculation with the initial values calculated from the nonlinear time-harmonic eddy-current analysis (starting from "jω" in Fig. 2), we can obtain steady-state solutions much faster than the case where the initial values in (3) are set to 0 (starting from 0 in Fig. 2).

Tables I compares the calculation time of the parallel TP-EEC method and TDPFEM in the steady-state analysis of the motor under PWM excitation. The number of time steps per period is 1440. All the computations were performed on a supercomputer Cray CS400 2820XT [10]. For comparison, we perform the sequential calculation with the simplified TP-EEC method as shown in Table I. We judge that the steady-state solutions are obtained when all the relative errors of torque and eddy-current loss in a half period are less than 3 % compared with the reference solutions. The parallel TP-EEC method can achieve better performance than the TDPFEM, which indicates the effectiveness of the proposed method. The detail of the formulation of the proposed method and more numerical results will be included in the full paper.

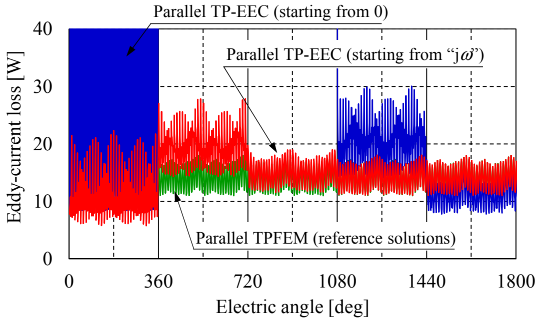

Fig. 2. Time variation of eddy-current loss in the secondary conductor under sinusoidal excitation at s = 0.588 (m=2, r=1, q=0).

TABLE I
Calculation Time under PWM Excitation

Number of processes	Parallel TP-EEC method: Calculation time [s]	Parallel TP-EEC method: Parallel speedup	TDPFEM: Calculation time [s]	TDPFEM: Parallel speedup
36	439.7	11.9	4157.5	1.25
72	318.8	16.4	2208.4	2.36
144	227.7	22.9	1187.9	4.39
360	312.5	16.7	531.3	9.81
sequential	5215.5	1.0	-	-

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