Adaptive Subdomain Model Order Reduction with Discrete Empirical Interpolation Method for Nonlinear Magneto-Quasi-Static Problems

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This paper presents a novel adaptive subdomain model order reduction (MOR) based on proper orthogonal decomposition (POD) and discrete empirical interpolation (DEI) methods for nonlinear magneto-quasi-static (MQS) problems. In this method, the nonlinear region is subdivided into two regions where one of region includes all those finite elements which have particularly strong saturation in the nonlinear material and the other region does not. MOR based on POD and DEI methods is applied only to the latter region. Both regions are determined from the previous solution automatically because the finite elements which have the strong nonlinearity in the ferromagnetic material may change at each time step. It is shown that this method can effectively reduce the computational time to solve the nonlinear MQS problems without losing quality of accuracy in comparison with the accuracy of the non-reduced finite element method.

***Index Terms*— Finite element analysis, principal component analysis, reduced order systems.**

I. Introduction

Model order reduction (MOR) has been applied to a variety of discrete electromagnetic field problems, for example for the transient analysis for electro- and magneto-quasi-static (EQS and MQS) problems [1]-[5]. Although we can apply MOR based on proper orthogonal decomposition (POD) to nonlinear problems, this method may not be remarkably effective for analyzing the nonlinear problems [1][2]. There are two problems in POD-based MOR: one is a deterioration of the accuracy in saturated regions and another is an increase of computational cost in iterative processes in which a nonlinear term has to be updated and matrix-matrix products have to be calculated at each time step.

To tackle the deterioration of the accuracy, one of the authors has proposed Subdomain MOR [3] in which the whole region is subdivided into linear and nonlinear regions. Then MOR is applied only to the linear region. This approach helps to maintain the accuracy in the saturated region compared to the non-reduced model. However, this method cannot dramatically reduce the computational time because the number of degrees of freedom (DoF) is much larger than that of the usual approach where a MOR is applied to the whole computational domain.

On the other hand, MOR based on discrete empirical interpolation (DEI) method [4][5] has been proposed to reduce the computational cost in the iterative processes. In this method, the nonlinear term is interpolated in the localized nonlinear regions which are determined by evaluating nonlinear terms obtained during a preconditioning process.

In this paper, we propose a novel POD-based MOR method combining Subdomain MOR and DEI methods in which a nonlinear region is subdivided into two regions. One has finite elements which exhibit a particularly strong nonlinear behavior in the ferromagnetic material while the other does not have them. POD-based MOR is applied only to the latter region. These regions should be determined automatically because the electromagnetic distributions in the nonlinear ferromagnetic material change at each time step and it is initially not known where the elements which exhibit strong nonlinearity due to ferromagnetic saturation effects are positioned before analyzing the electromagnetic distributions. Therefore, these regions are automatically determined from the previous solutions and the snapshot electromagnetic distributions. Moreover, then the DEI method is applied to the nonlinear term in the latter region to reduce the computational time in the iterative processes.

In this work, we apply the present method to two models for MQS problems and compare the present method to usual MOR, DEI-based MOR and Subdomain MOR with respect to the accuracy and the computational time.

II. Formulation

Let us consider a nonlinear equation for MQS problems of n dimension,

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{\mathrm{sub}} \\ \boldsymbol{x}_{\mathrm{MOR}} \\ \boldsymbol{x}_{\mathrm{linear}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{b}_{\mathrm{sub}} \\ \boldsymbol{b}_{\mathrm{MOR}} \\ \boldsymbol{b}_{\mathrm{linear}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_{\mathrm{sub}}(\boldsymbol{x}) \\ \boldsymbol{f}_{\mathrm{MOR}}(\boldsymbol{x}) \\ 0 \end{bmatrix} = 0 \tag{1}$$

where the first and second terms are linear terms, the third term is a nonlinear term, $\boldsymbol{x}_{\mathrm{sub}} \in \mathrm{R}^{(n-p-q)}$, $\boldsymbol{x}_{\mathrm{MOR}}(\boldsymbol{x}) \in \mathrm{R}^{p}$, $\boldsymbol{x}_{\mathrm{linear}} \in \mathrm{R}^{q}$, p and q are the number of DoFs in the domains to which MOR is applied in nonlinear and linear domains, respectively. Applying the singular value decomposition to $\mathrm{X}_{\mathrm{MOR}}=[\boldsymbol{x}_{\mathrm{MOR}}(t_1),\ldots, \boldsymbol{x}_{\mathrm{MOR}}(t_s)]$ and $\mathrm{X}_{\mathrm{linear}}=[\boldsymbol{x}_{\mathrm{linear}}(t_1),\ldots, \boldsymbol{x}_{\mathrm{linear}}(t_s)]$, we can obtain transformation matrices $\mathrm{W}_{\mathrm{MOR}}$ and $\mathrm{W}_{\mathrm{linear}}$. The vectors $\boldsymbol{x}_{\mathrm{MOR}}$ and $\boldsymbol{x}_{\mathrm{linear}}$ can be expressed by linear combination of the transformation matrices $\mathrm{W}_{\mathrm{MOR}}$ and $\mathrm{W}_{\mathrm{linear}}$

$$\boldsymbol{x} = \mathrm{W}\boldsymbol{y} = \begin{bmatrix} \mathrm{I} & 0 & 0 \\ 0 & \mathrm{W}_{\mathrm{MOR}} & 0 \\ 0 & 0 & \mathrm{W}_{\mathrm{linear}} \end{bmatrix} \boldsymbol{y} \tag{2}$$

where I is an unit matrix, s is the number of snapshots and $\boldsymbol{y}\in R^{p+s+s}$. Moreover, we can apply DEI method to $\boldsymbol{f}_{\mathrm{MOR}}(\boldsymbol{x})$ to reduce the computational time in the iterative processes, i.e.

$$\boldsymbol{f}_{\mathrm{MOR}}'(\boldsymbol{x}) = \mathrm{U}(\mathrm{P}^t\mathrm{U})^{-1}\mathrm{P}^t \boldsymbol{f}_{\mathrm{MOR}}(\boldsymbol{x}) \tag{3}$$

where U and P are found in [4]. Finally, we can obtain the reduced equation for (1)

$$\mathrm{W}^t\mathrm{A}\mathrm{W}\boldsymbol{y} + \mathrm{W}^t\boldsymbol{b} + \mathrm{W}^t\mathrm{W}_{\mathrm{DEI}}\boldsymbol{f}(\mathrm{W}\boldsymbol{y}) = 0\,, \tag{4}$$

where

$$\mathrm{W}_{\mathrm{DEI}} = \begin{bmatrix} \mathrm{I} & 0 & 0 \\ 0 & \mathrm{U}(\mathrm{P}^t\mathrm{U})^{-1}\mathrm{P}^t & 0 \\ 0 & 0 & \mathrm{I} \end{bmatrix} \tag{5}$$

We employ the finite element method and Newton Raphson method to solve (4) for the nonlinear MQS problems.

III. Adaptive subdomain MOR

In the preconditioning process in which we snapshot the electromagnetic field until time step s, the nonlinear region is automatically subdivided into two regions by using the norm of the magnetic density and we construct the transformation matrices W and $\mathrm{W}_{\mathrm{DEI}}$ at each time step. At n time step, the electromagnetic distribution which does not significant change from n-1 time step is detected from the magnetic distributions obtained in the preconditioning process. The transformation matrices corresponding to this detection are chosen from the trans-formation matrices constructed in the preconditioning process.

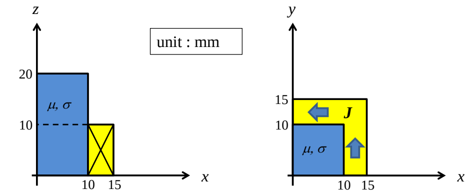
IV. Results

To test the validity of the present method, we apply usual MOR, DEI-based MOR, subdomain MOR and the present method to two models for the MQS problems shown in Fig. 1 which are connected to the simple circuit shown in Fig. 2 where v and R are 0.008V and $10^{-5}\Omega$. The driving frequency is 100Hz and Δt is 2.5×10^{-4} seconds. The initial states are assumed to be zero fields. The number of snapshots is set to s=20 and 50 in model 1 and 2, respectively. To compare the present method to the other methods, the error is defined by

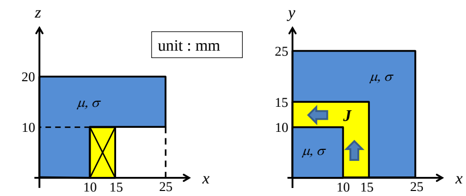
$$\mathrm{Error} = \frac{1}{N_t}\sum_{n=1}^{N_t}\sqrt{\frac{\sum_{k=1}^{N_e}\left|\boldsymbol{B}_{k,n} - \boldsymbol{B}_{k,n}^{\mathrm{MOR}}\right|^2}{\sum_{k=1}^{N_e}\left|\boldsymbol{B}_{k,n}\right|^2}} \tag{6}$$

where N_t, N_e, $\boldsymbol{B}_{k,n}$ and $\boldsymbol{B}_{k,n}^{\mathrm{MOR}}$ are the total time steps, the number of elements, the magnetic density for k-th element at n time step obtained by FEM and each MOR method. The computational time does not include the preconditioning process in which we snapshot the electromagnetic fields and construct transformation matrix and so on.

The results of the errors and computational times are shown in TABLE I and II. The DEI-based MOR performs best and worst with respect to the computational time and the error, respectively, while the Subdomain MOR behaves just the opposite in both models. These methods have trade-off property between the error and the computational time. In model 2, the computational times of the MOR and Subdomain MOR are more than 100% of the standard FEM simulation. Here, in the case of nonlinear MQS problems, these methods have to calculate the matrix-matrix products at each time step whose computational cost increases with number of snapshots. We can see in TABLE I and II that the errors of the present method are almost the same as those of subdomain MOR while the computational times of the present method are less than 30% in both models. Thus, the presented method can improve accuracy and computational time, simultaneously.



(a) Model 1



(b) Model 2

Fig. 1 Numerical model

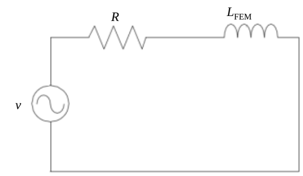


Fig. 2 Circuit including FE model

TABLE I

Computational cost obtained by each MOR method.

	MOR	SubMOR	MOR_DEI	Present
Model1	23.2%	40.9%	8.90%	13.7%
Model2	145%	118%	8.14%	27.7%

TABLE II

Error obtained by each MOR method.

	MOR	SubMOR	MOR_DEI	Present
Model1	3.63%	0.289%	3.82%	0.474%
Model2	3.45%	0.749%	5.47%	0.982%

The full paper will give details on the automatic subdivision of the nonlinear region and the effectiveness of this method.

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