Research on Cogging Torque Calculation for Interior Permanent Magnet Machine based on Lumped-Circuit Parameters

Jangho Seo[1] and Hong Soon Choi[2]
[1]School of Automotive Engineering, Kyungpook National University, Sangju, 742-711, Korea
[2]Department of Electrical Engineering, Kyungpook National University, Daegu, 702-711, Korea
tochs@knu.ac.kr

***Abstract*—This paper is the first research showing that it is possible and effective to predict cogging torque waveform of interior permanent machine (IPM) using only analytical model. Based on proposed virtual permanent magnet concept, interior permanent magnet can be considered as surface permanent magnet having zero height and unit relative permeability, and then it is used for calculating the cogging torque with conventional techniques. The proposed method is validated by finite element analysis (FEA).**

***Index Terms*—Analytical model, cogging torque, interior permanent magnet machine.**

I. INTRODUCTION

Cogging torque is defined as a torque produced by magnetic forces between stator teeth and the magnets of rotor, and it can be an important performance index of acoustic noise and vibration. Thus, it is generally desirable to reduce the cogging torque at design stage.

Regarding analysis models for the torque, there are several techniques for surface permanent magnet machine (SPM) based on analytical approaches which is useful and practical in terms of their calculating time compared to FEA. In the case of IPM, however, it is difficult to take magnetic saturation and complex rotor structure into account in space harmonic analysis method[1]. Thus, numerical approach such as FEA has been inevitably used, which should be very time consuming [1]-[3].

In this paper, a simple analytical technique based on both lumped-circuit and existing cogging torque model, is proposed to synthesize the cogging torque waveform in IPM.

II. ANALYSIS MODEL AND LUMPED MAGNETIC CIRCUIT

Fig. 1 shows the IPM machine with 'V' shape permanent magnet (PM) having internal and external bridges.

The lumped models for open-circuit field is shown in Fig. 2. α_P is the pole-arc to pole-pitch ratio and N_P is the number of magnet poles. R_{mb1} and R_{mb2} are the reluctances of internal and external bridges and the corresponding flux are ϕ_{mb1} and $\phi_{m\,b2}$ which are represented by flux line as shown in Fig. 1.

ϕ_r and ϕ_{mo} are the flux source and the leakage flux of PM. ϕ_{1ml} and ϕ_{2ml} are the magnet end-leakage fluxes and the corresponding reluctances are R_{1ml} and R_{2ml}. R_s and R_r are the reluctances of the stator yoke and rotor yoke. Detail expression for the lumped parameters can be found in [4].

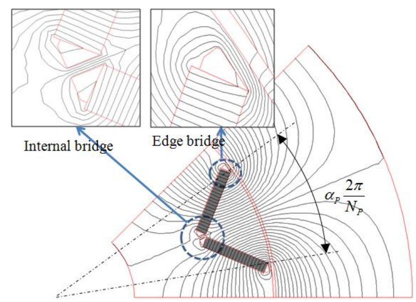

Fig. 1. IPM machine structure with 'V' shape magnet

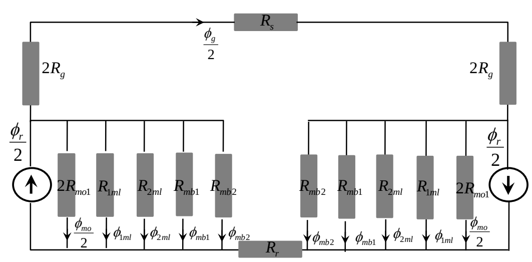

Fig. 2. Lumped magnetic circuit for analysis model

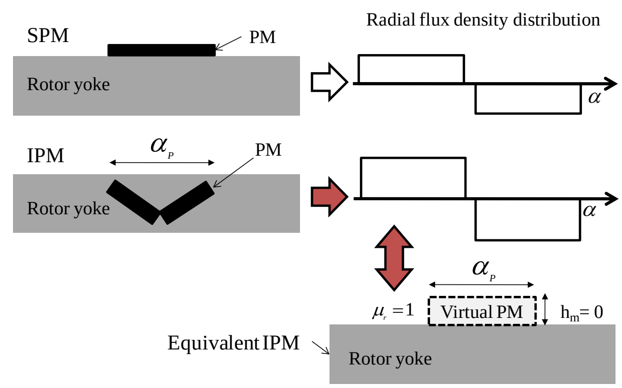

Fig. 3. Virtual PM concept for cogging torque calculation

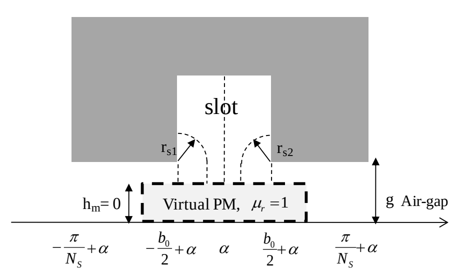

Fig. 4. Approximation of flux line paths in a slot of IPM for *Model I*

III. Proposed Cogging Torque Calculation Method

In order to combine lumped-circuit model with the conventional cogging torque techniques which consider only radial flux density waveform, a new virtual PM (VPM) concept is proposed. The illustration of the idea is represented in Fig. 3.

With respect to radial flux density distribution, the pattern of SPM and IPM having 1-layer magnet are similar. Thus, we can regard the IPM structure as SPM having zero height and unit relative permeability of PM while it still has same radial flux density and the ratio of pole-arc to pole pitch of IPM. Therefore, it is possible to predict the cogging torque of IPM with existing cogging torque models used for SPM.

It is worth noting that this concept is only available for radial air-gap flux density. In addition, it is not necessary to calculate equivalent residual-flux density (B_r) of the VPM because operating point of the magnet was already considered in lumped-circuit model. Indeed, the cogging torque calculation model used in this report do not utilize the B_r but use only flux density of air-gap predicted by lumped-circuit model. Some of techniques, two models for the concept are selected.

A. *Model I*

The cogging torque can be calculated analytically from the net lateral force acting on the stator teeth[3]:

$$T_{cog}(\alpha) = \frac{L_{ef}}{2\mu_0}\left[\int_0^{r_{s1}} \left(B(\alpha)\tilde{\lambda}_{s1}\right)^2 \left(R_s + r_{s1}(\alpha)\right) d\alpha - \int_0^{r_{s2}} \left(B(\alpha)\tilde{\lambda}_{s2}\right)^2 \left(R_s + r_{s2}(\alpha)\right) d\alpha\right] \tag{1}$$

where *B(α)* is gained from the lumped-circuit parameters. Thanks to the virtual PM, the permeance and the relative permeance for the flux of PM in the slot can be simplified into

$$\lambda = \frac{\mu_0}{g + \dfrac{h_m}{\mu_r} + \dfrac{2\pi r_s}{4}} = \frac{\mu_0}{g + \dfrac{2\pi r_s}{4}}, \quad \tilde{\lambda} = \frac{\lambda}{\dfrac{\mu_0}{g + \dfrac{h_m}{\mu_r}}} = \frac{\lambda}{\dfrac{\mu_0}{g}}. \tag{2}$$

The illustration of *Model I* is represented in Fig. 4.

B. *Model II*

By supposing that energy variation in PMs and iron is negligible compared to that of air-gap, the cogging torque can be expressed as follows[5]:

$$T_{cog}(\alpha) = \frac{\partial W_{airgap}(\alpha)}{\partial \alpha} = \frac{\partial}{\partial \alpha}\left[G^2(\theta) B^2(\theta, \alpha) dV\right] \tag{3}$$

where *G(θ)* is a modulation function called air-gap relative permeance.

IV. Simulation Results and Discussion

Fig. 5 shows comparison results of air-gap flux density waveform by the lumped-circuit and FEA. Based on (1), (2) and (3), the cogging torque waveform of IPM can be predicted and their results are shown in Fig. 6. In order to confirm the performance of *Model I* and *Model II* in reducing cogging torque of IPM, influence of pole-arc to pole pitch ratio was investigated as shown in Fig. 7. From the analysis results, it is shown that *Model I* shows better performance results compared to *Model II*. Moreover, *Model I* can find optimal point for minimum cogging torque as shown in Fig. 7.

References

[1] H. S. Ko and K. J. Kim, "Characterization of noise and vibration sources in interior permanent magnet brushless DC motors," *IEEE Trans. on Magn.*, vol. 40, no. 6, pp. 3482-3489, 2004.

[2] C. Bianchini, F. Immovilli, E. Lorenzani, A. Bellini, and M. Davoli "Review of design solutions for internal permanent-magnet machines cogging torque reduction, " *IEEE Trans. on Magn.*, vol. 48, no. 10, pp. 2685-2693, 2012.

[3] Z. Q. Zhu and D. Howe, "Analytical prediction of the cogging torque in radial-field permanent magnet brushless motors," *IEEE Trans. on Magn.*, vol. 28, no. 2, pp. 1371-1374, 1992.

[4] Li Zhu, S. Z. Jiang, Z. Q. Zhu, and C. C. Chan, "Analytical modeling of open-circuit air-gap field distributions in multisegment and multilayer interior permanent-magnet machines, " *IEEE Trans. on Magn.*, vol. 45, no. 8, pp. 3121-3130, 2009.

[5] S. M. Hwang, J. B. Eom, Y. H. Jung, D. W. Lee, and B. S. Kang, "Various design techiniques to reduce cogging torque by controlling energy variation in permanent magnet motors, " *IEEE Trans. on Magn.*, vol. 37, no. 4, pp. 2806-2809, 2001.

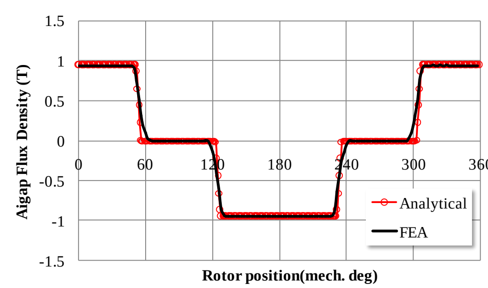

Fig. 5. Comparison of predicted air-gap flux density distribution

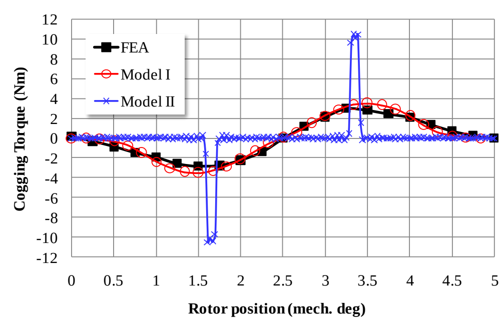

Fig. 6. Waveform for cogging torque

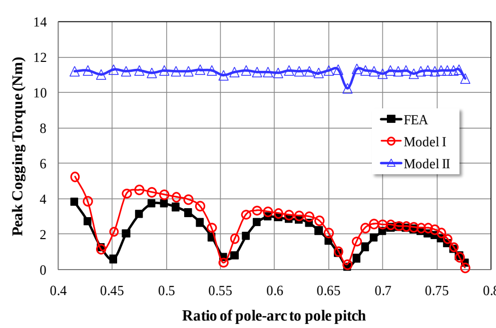

Fig. 7. Influence of pole-arc to pole pitch ratio on peak cogging torque