An Improved Analytical Method for Calculation of PMEM Cogging Torque

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***Abstract*— In this paper, an improved analytical method for calculations of magnetic field distribution and cogging torque of surface-mount permanent magnet electrical machine (PMEM) is presented. The new method takes into account the magnetic saturation and flux leakage of PMEMs, leading to improved accuracy while retaining fast computation. The proposed method has been validated through both the finite element analysis and tests using two 10kW PMEMs for variable speed wind turbines.**

***Index Terms* — Permanent magnet electrical machines, equivalent circuits, finite element method, torque.**

I. INTRODUCTION

As more permanent magnet electrical machines (PMEMs) are applied in wind and marine energy systems, the need for high performance PMEMs has demanded improved design methodologies, particularly faster and more accurate calculation methods for machine parameters. Cogging torque is an important PMEM design parameter, and its calculations are often time-consuming [1],[2]. Analytical method, being simple, fast and practical, is widely used in PMEM design.

Most of the existing analytical methods for cogging torque calculations follow two steps: first, deriving the position-dependent flux density distribution in the air gap, and then calculating the cogging torque by either the co-energy method or the electromagnetic force method [3],[4]. These traditional analytical methods for cogging torque calculation are all based on the assumptions of infinite iron permeance and negligible leakage flux. However, saturation and flux leakage are very common. Thus, these simplifying assumptions lead to significant errors. An improved analytical method for cogging torque calculation has been developed by the authors with the help of equivalent magnetic circuits. With magnetic saturation and flux leakage being taken into account, the new analytical method leads to more accurate calculations of cogging torque.

II. ANALYTICAL METHOD FOR COGGING TORQUE CALCULATIONS

The scalar magnetic potential φ in the area between the stator inner bore and rotor outer surface of a PMEM is governed by the Laplace/quasi-Poissonian field equations [5]:

$$\begin{cases} \dfrac{\partial^2\varphi_I}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial\varphi_I}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial\varphi_I^2}{\partial\theta^2}=0 & \text{in air gap area} \\ \dfrac{\partial^2\varphi_{II}}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial\varphi_{II}}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial\varphi_{II}^2}{\partial\theta^2}=\dfrac{M_r}{r\mu_r} & \text{in magnet area} \end{cases} \tag{1}$$

The flux density distribution B_{PM} can be solved analytically from (1). By summing the lateral magnetic forces along the walls of stator teeth, the cogging torque can be derived [6]:

$$T_{cog}(\theta)=\frac{\pi L R_s}{2\mu_0 N}\sum_{m=1}^{N}\left[B_{PM}^2\left(\frac{2\pi}{N}m+\theta\right)\left(R_m+g_\alpha\right)\cdot ssg\right] \tag{2}$$

III. PMEM EQUIVALENT MAGNETIC CIRCUIT

Fig. 1 shows the equivalent magnetic circuit of a PMEM at no-load condition. φ_m is the total magnet flux composing of the magnet leakage flux φ_σ and the effective flux φ_δ.

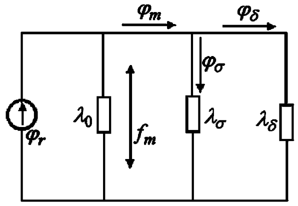

Fig. 1. PMEM equivalent magnetic circuit (no-load)

The total magnet external permeance λ_n is composed of the magnet leakage permeance λ_σ and the air gap λ_g plus machine iron permeance λ_δ. Due to saturation, λ_n is a curve as shown in Fig. 2. φ_δ can be solved through an iterative process by a computer simulation tool.

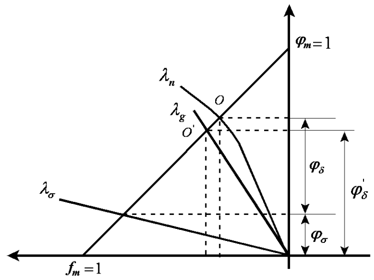

Fig. 2. Magnet demagnetization curve of PMEM

IV. IMPROVED ANALYTICAL METHOD FOR CALCULATION OF PMEM COGGING TORQUE

An improved analytical method has been developed by the authors for cogging torque calculations. The method combines the PMEM equivalent magnetic circuit in Section III with the traditional analytical calculation method in Section II, in order to compensate for the inaccuracy caused by neglecting the PMEM saturation and flux leakage by the traditional analytical method for cogging torque calculations.

Since both the magnet leakage flux and the PMEM saturation are neglected in the analytical method of Section II, the calculated air gap flux is φ_δ' as shown in Fig. 2, which accounts only the air gap permeance λ_g and is bigger than the actual effective air gap flux φ_δ. A correction factor K_c is introduced to account for the effects of both the magnetic saturation and leakage flux,

$$K_c = \frac{\varphi_\delta}{\varphi'_\delta} = \frac{\lambda_n\left(\lambda_g+1\right)}{\sigma_0\lambda_g\left(\lambda_n+1\right)} \qquad (3)$$

where σ_0 is the flux leakage coefficient which can be determined based on the PMEM geometry [7].

The correction factor K_c is used to adjust the flux density B_{PM} calculated from the traditional analytical method, and then (2) is used to calculate the cogging torque of PMEM.

Fig. 3 shows the air gap flux density distributions calculated using the traditional analytical method (B-Anal), improved method proposed in this paper (B-Modi), and finite element method (B-FEM). The PMEM used in the calculations is a 3-phase 10kW permanent magnet synchronous generator (PMSG) for direct drive wind turbines.

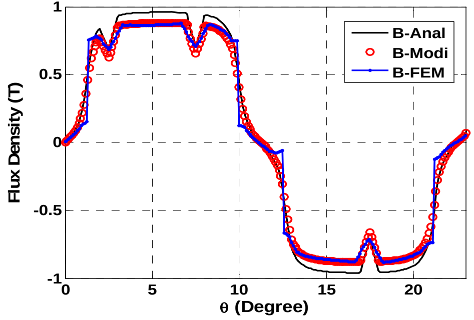

Fig. 3. Calculated flux density from different methods

Fig. 3 reveals that the flux density from the traditional analytical method exhibits larger errors due to its negligence of both the saturation and flux leakage. Using correction factor K_c, the improved analytical method provides an accurate flux calculation in a practical manner, as verified by the FEM results. This leads to an improved calculation for cogging torque of PMEMs as presented in Section V.

V. VALIDATION OF COGGING TORQUE CALCULATIONS

The proposed improved analytical method for calculation of PMEM cogging torque has been used to design and investigate the cogging torque of two PMSGs for direct drive small wind turbines: one is a 3-phase 10kW PMSG, and the other is a 2-phase 10kW PMSG

Fig. 4 and Fig. 5 show the calculated cogging torque of the 2-phase and 3-phase PMSGs using the proposed improved analytical method.

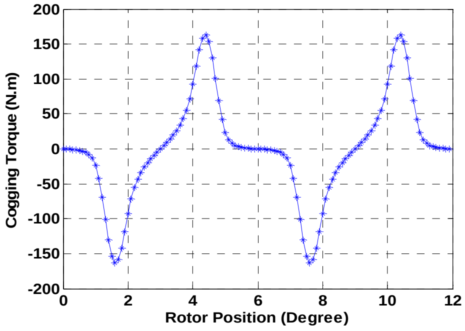

Fig. 4. Cogging torque of 2-phase PMSG

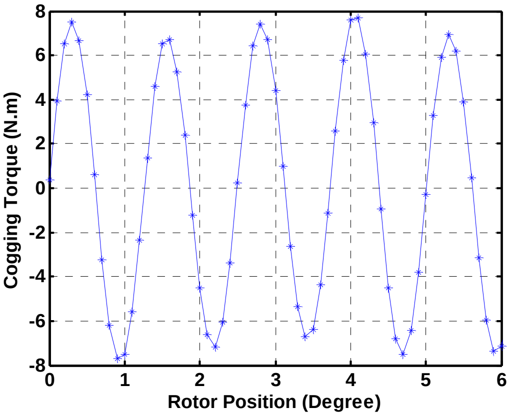

Fig. 5. Cogging torque of 3-phase PMSG

Table I presents the comparison of calculated cogging torque with both the FEM results and test results.

TABLE I
PEAK COGGING TORQUE (N-M)

Methodologies	3-phase PMSG	2-phase PMSG
Analytical Method	7.9	165
FEM	7.1	172
Test	6.8	163

The comparative results have demonstrated that the proposed analytical method offers a practical, fast and accurate calculation of cogging torque with the saturation and leakage of PMEMs being taken into account. The proposed method can be readily applied in calculations of other parameters, such as PMEM torque ripple and electromotive force etc.

VI. CONCLUSIONS

An improved analytical method has been developed by the authors and applied for calculations of cogging torque of surface-mount permanent PMEMs. The new method takes into account the magnetic saturation and flux leakage of PMEMs and thus provides more accurate results as compared with traditional analytical methods. The proposed analytical method has been validated through both the finite element method (FEM) and tests of two wind turbine PMEMs.

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