Fast Computation of Torque - Load Angle Characteristics of Synchronous Machines Using Time-Domain Finite Element Method

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***Abstract* — Synchronous machines have wide applications in many areas including industrial, agricultural, and domestic. Almost all alternating current (ac) generators are synchronous machines. The torque - load angle relationship is an essential characteristic to determine the behavior of the synchronous machines. Conventionally, for each load angle, a transient magnetic field using time-domain finite element method (FEM) needs to be solved, which is very time-consuming to obtain the whole characteristic. In this paper, a fast variable phase method for computing the torque - load angle characteristic of the synchronous machines is presented, which only needs to computed one transient process of the time-domain FEM. The proposed method only needs 20% of the computing time of the conventional method.**

***Index Terms*—Finite element method, synchronous machine, time-domain, torque – load angle characteristic.**

I. Introduction

Synchronous machines are one of important electromechanical energy converters [1]. They have wide applications both in constant speed drives as motors and in variable speed controls when interface to the power sources with variable frequency converter systems. It is also used in alternators that generate the power for automobile electrical systems. The electromagnetic torque T_{em} is determined by the two magnetic fields, which are the stator field and the rotor field, and the space angle difference between the two fields. The electrical angle δ between the stator and rotor fields is commonly called the load angle. The output power of the machines is proportional to the output torque, so the output power - load angle characteristic of the synchronous machines can be presented by the electromagnetic torque T_{em} - load angle δ characteristic. The torque - load angle characteristic, therefore, is essential to determine the behavior of the synchronous machines.

The time-domain finite element method (FEM) has become an effective tool to analyze the synchronous machines [2-4]. In order to obtain the torque - load angle characteristic, the synchronous machines are analyzed using the time-domain FEM to achieve the steady-state solution under each load angle. It means that the output torque should become stable at each time with one specific load angle. This time consuming computation needs to repeat many times before the complete curve of the torque versus the load angle is found.

In this paper a variable phase computation method for obtaining the torque - load angle curve is presented. A numerical experiment shows that it can significantly reduce computation time.

II. Methods

A. Variable Phase Method

The voltages applied to the terminals of the three-phase windings are:

$$\left.\begin{aligned} v_A &= V_m \sin(\omega t + \phi_0) \\ v_B &= V_m \sin(\omega t + \phi_0 - 2\pi/3) \\ v_C &= V_m \sin(\omega t + \phi_0 - 4\pi/3) \end{aligned}\right\} . \quad (1)$$

The initial rotor's position is adjusted so that the initial phase ϕ_0 is equal to the load angle δ.

A convenient method is proposed here to estimate whether the solution is near steady-state. During steady-state operation for problems, the average value of the torque over time period can be used as an indicator of whether the solution is close to its steady-state. At time t, the average value of the electromagnetic torque $T_{em}(t)$ over the preceding period T can be calculated as a time function

$$T_{em(av)}(t) = \frac{1}{T}\int_{t-T}^{t} T_{em}(t)\mathrm{d}t \,. \quad (2)$$

If $|T_{em(av)}(t)|$ is almost constant, the solution is assumed to have reached its steady-state.

B. Nonlinear Iteration Formulation

In the FEM of magnetic field computation, the nonlinear N-R iterative method is applied into the field equation and circuit equations in the magnetic field regions. During nonlinear iteration, if nonlinear iteration error R is smaller than the error tolerance e_t the iteration will stop. Because only the steady-state solution is sought, a relatively rough solution during the transient process will suffice. The tolerance of nonlinear iteration can start from a large value before it is gradually reduced to a nominal value, $e_t = 10^{-f}$, and $f = f_{\text{initial}} + (f_{\text{normal}} - f_{\text{initial}})t/t_s$, where; f_{initial} is the initial value when $t = 0$; f_{normal} is the normal value for steady-state solution. t_s is the estimated time to reach steady-state of operation.

C. Adjustment of Time Step Size

During the transient process, the time step size Δt for time integration is automatically adjusted gradually and smoothly from a large value to a regular value until the steady-state solution is reached. At each time step,

$$\Delta t = M \cdot (\Delta t)_s \left(1 - \frac{M-1}{M}\frac{t}{t_s}\right), \quad (3)$$

where $(\Delta t)_s$ is the time step size when the transient process reaches its steady-state. In (3) the time step size starts from a large value of M times of the normal step size, and then it is linearly reduced to its normal value.

III. Numerical Example

An octupole nine slot permanent magnet (PM) synchronous machine as shown in Fig. 1 is used to showcase the proposed method. The detailed specifications are listed in Table I and its typical flux distribution is shown in Fig. 2.

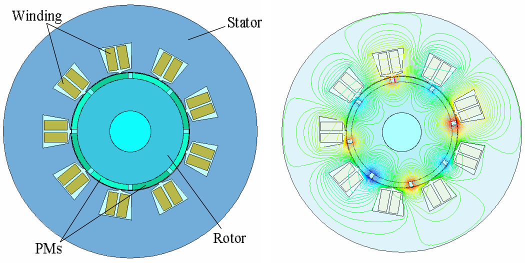

Fig. 1. Overview of an intersection structure of a PM synchronous machine and a flux plot on an intersection of PM synchronous machine.

TABLE I Key Design Specifications of PM Synchronous Machine

Frequency	50 Hz
Number of phase	3
Axial length	65 mm
Outer radius of outer stator	80 mm
Inner radius of outer stator	37.5 mm
Outer radius of motor PM	37 mm
Inner radius of motor PM	33.5 mm
Inner radius of inner motor	13 mm
Number of stator slots	9
Number of stator pole pairs	4

A. *Fixed Time-stepping Size and nonlinear iteration error*

In the analysis, the time-stepping size is set to be a fixed value of 0.2 ms. The initial phase ϕ_0 will be changed to another value after the solution reaches a steady state. The output torque under different load angle varied with time is given in Fig. 2. Then the torque - load angle characteristic is obtained as shown in Fig. 3.

B. *Adaptive Time-stepping Size and nonlinear iteration error*

According to the above mentioned methods, the time-step size and nonlinear iteration error are adaptively adjusted during analysis process.

The output torque curve is shown in Fig. 4. The average output torque at each load angle almost matches with that in Fig. 2. The time computation time in this method can reduce about 20% compared with the fixed time step size method.

IV. Conclusion

In this paper, a fast variable phase method for computing the torque - load angle characteristic of the synchronous machines is presented, which only needs to computed one transient process of the time-domain FEM. From the results, it can be seen that the proposed method can obtain the same result compared with the conventional method. However, the proposed method only needs 20% of the computing time of the conventional method.

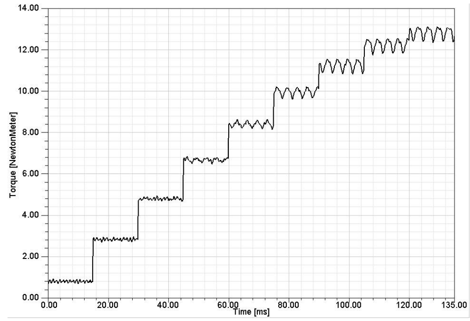

Fig. 2. The output torque curve with fixed time-step size and fixed nonlinear iteration error obtained by using time-domain FEM.

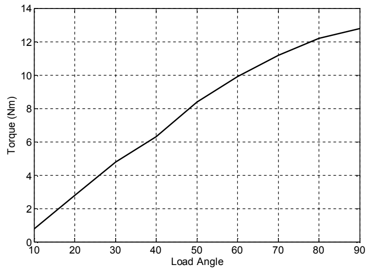

Fig. 3. Computed torque - load angle characteristic.

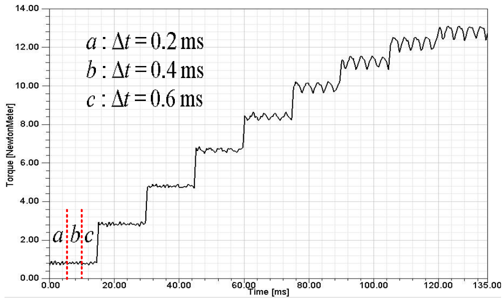

Fig. 4. The output torque curve with adaptive time-step size and adaptive nonlinear iteration error obtained by using time-domain FEM.

V. References

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