Transient Performance Analysis of Single-Phase Induction Motor by Using Field-Circuit Coupled Finite Element Method

Youpeng HuangFu[1], Shuhong Wang[1], Jie Qiu[1], Haijun Zhang[1], Guolin Wang[1], Jianguo Zhu[2]
[1]Faculty of Electrical Engineering, Xi'an Jiaotong University, Xi'an, 710049, China
shwang@mail.xjtu.edu.cn
[2]Faculty of Engineering and Information Technology, University of Technology, Sydney, NSW2007, Australia

Abstract—**In this paper, the application of finite element method (FEM) for predicting the performance of a single phase induction motor (SPIM) under a sinusoidal voltage-forced operation combining with VB and AutoCAD software is described. Numerical analysis is performed by solving the nonlinear time-stepping finite-element equation coupled with the magnetic field equation, circuit equation, and mechanical equation of motion, and the transient mathematic analytical model was built combined the method of electromagnetic field. An experiment using a resistance-starting, capacitor-running (RS-CR) SPIM is presented, and detailed analysis about its speed, current, torque, core-loss, back EMF and power factor are reported. Good agreement is achieved between calculated and test results.**

Index Terms—**Single Phase Induction Motor, Field-Circuit Coupled, Finite Element Method**

I. INTRODUCTION

Time-stepping finite element method (TS-FEM), which couples magnetic field equations with external electric circuit equations and mechanical torque balance equations, has been successfully used to analyze the transient performance of motors. Effects of skin effect, iron-core loss, non-linearity and mechanical movement of the rotor can all be taken into consideration [1]-[2]. E.VASSENT used a step by step FEM in induction motor simulation [3]. C.-G. Hong used nonlinear complex current source FEM in induction motor analysis [4]. Xiuhe Wang used the analytic method based on equivalent circuit to analyze the performance of SPIM [5]. S. L. Ho used TS-FEM to analysis permanent magnet machine [6].

In this paper, the voltage source TS-FEM analytical method is presented. While, the end leakage inductance for main and aux. windings, end ring resistance, end ring leakage inductance are calculated using Magnetic Circuit Method. The transient performance of a RS-CR SPIM is studied. Its speed, current, torque, core-loss, back EMF and power factor of the motor are analyzed.

II. METHOD OF ANALYSIS

A. Magnetic field equations

The electromagnetic field equation of a induction motor in two-dimensional (2-D) Cartesian coordinates can be described as follows:

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_Z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_Z}{\partial y}\right) = -J_S+\sigma\left(\frac{\partial A_Z}{\partial t}-\vec{v}\times\boldsymbol{B}-\nabla\varphi\right) \quad (1)$$

where μ, A_Z, J_S, σ, $\vec{v}$ and φ are the magnetic permeability, the magnetic vector potential in z component, the stator current density, the conductivity of rotor bar, the velocity of the rotor and the electric scalar potential, respectively.

Assuming that the solution region is meshed with triangular elements, then applying the Galerkin FEM, the nonlinear matrix equation can be expressed as follows:

$$\boldsymbol{KA} = \boldsymbol{P} \quad (2)$$

where $\boldsymbol{K}$, $\boldsymbol{A}$ and $\boldsymbol{P}$ are the coefficient of stiffness matrix, magnetic vector column, the right side matrix.

B. Circuit equations

The equation of the stator winding circuit is given as:

$$U_S = R_M I_M + L_M\frac{dI_M}{dt}+\frac{d\Psi_M}{dt} \quad (3)$$

$$U_S = R_A I_A + L_A\frac{dI_A}{dt}+\frac{d\Psi_A}{dt}+\frac{1}{C}\int I_A dt \quad (4)$$

where U_S, R_M, R_A, I_M, I_A, L_M, L_A, Ψ_M and Ψ_A are the input voltage, the resistance of coil, input current, end-turn inductance, and flux linkage of the main and aux. windings, respectively.

The equivalent rotor circuit is considered as a multiphase circuit shorted by the end rings. Based on Kirchhoff's law, the loop equation of the *n*th rotor bar can be written as:

$$I_{b_n} = \frac{U_{b_{n+1}}-U_{b_n}}{2Z_{e_n}}+\frac{U_{b_{n-1}}-U_{b_n}}{2Z_{e_{n-1}}} \quad (5)$$

where U_{bn}, I_{bn} and Z_{en} are the induced voltage, the current of the *n*th rotor bar and the impedance of an end-ring segment, respectively.

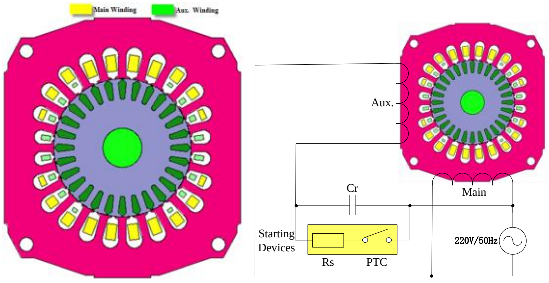
C. Mechanical equation

The mechanical equation of the shaft can be determined by using Newton's equation:

$$J\frac{d\omega}{dt} = T_e - k\omega - T_l \quad (6)$$

where J, k, T_e and T_l are the inertial momentum of rotor, the friction const, the electromagnetic torque and the load torque, respectively.

III. RESISTANCE-STARTING, CAPACITOR-RUNNING SPIM



(a) (b)

Fig. 1. Prototype SPIM and stator winding connection.
(a) SPIM. (b) Winding connection.

The configuration and winding connection of a prototype RS-CR SPIM are shown in Fig. 1(a), (b) respectively. As shown in Fig. 1, both consist of main and aux. windings in the stator and conductor bars to produce the starting torque in the rotor. Starting resistance Rs, running capacitor Cr, and positive temperature coefficient (PTC) are connected with the auxiliary winding to increase the starting torque and power factor.

IV. PERFORMANCE ANALYSIS AND DISCUSSION

Fig. 2 shows the process of SPIM finite element analysis based on VB and Auto-CAD software.

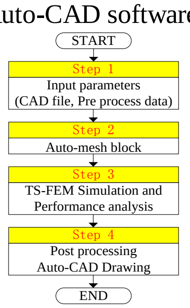
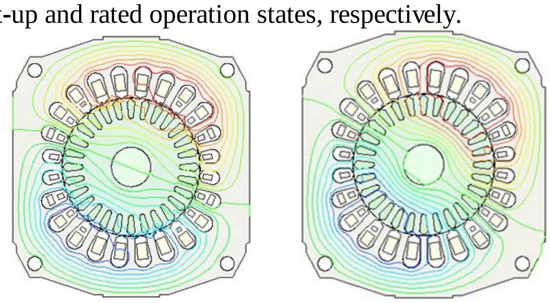


Fig. 2. Flowchart of design procedure

Once the magnetic field is determined by the nonlinear TS-FEM, the stator current is then determined by using the fixed voltage source and the external and inner impedances through iterative procedures, the electromagnetic torque is calculated by using the Maxwell stress tensor. Then mechanical motion is solved by the Runge-Kutta method. After the mechanical equation determines a new angular and radial position of a rotor, the finite element model is rearranged by moving mesh technique to recalculate the magnetic field.

Fig. 3(a), (b) shows the magnetic flux distribution of start-up and rated operation states, respectively.



(a) (b)

Fig. 3 . Magnetic flux distribution.
(a) Start-up state. (b) Rated operation state.

Based on the aforementioned AutoCAD software, performance analysis for the given SPIM is performed. Fig. 4 (a), (b), (c), (d), (e), (f), show the transient performance curves, the voltage source, stator current, transient speed, electromagnetic torque, core-loss, back EMF, respectively.

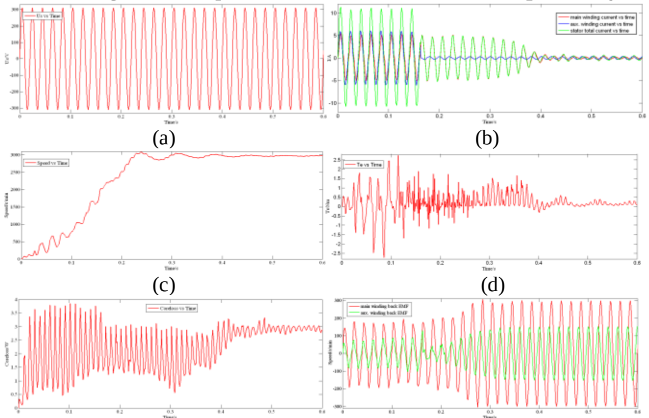


Fig. 4. Transient performance curves.
(a) Voltage source. (b) Stator current. (c) Speed. (d) Electromagnetic torque. (e) Core-loss. (f) Back EMF.

Table I show the main performance comparison between calculated and test results. The electromagnetic torque (T_e), the current of voltage source (I_S), the current of main winding (I_M) and the aux. winding (I_A), and the rated speed is got respectively. Pf and EFF are values of power factor and efficiency.

TABLE I
PERFORMANCE CALCULATED VS TESTED

Performance	Test Value	Calculation Value	Unit
EFF	83.46	82.29	%
Pf	0.86	0.8116	
I_S	0.32	0.3381	A
I_M	-	0.2635	A
I_A	-	0.2313	A
T_e	-	0.2093	N m
T_l	0.16	0.16	N m
Speed	2961	2965	r/min
Iron core-loss	-	2.9017	W

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