

Analytical Model of Induction Motor for Performance Calculation

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Abstract—This paper presents an analytical model of Induction motor (IM) for performance calculation. The model calculates the motor performance using air gap field based on geometric data. A 2D analysis based on polar coordinates is used to calculate the instantaneous air gap field waveform. The air gap field waveform is derived from Laplacian equation in air gap in polar coordinates. The effect of slotting is considered by including relative permeance function in the model. The motor parameters such as rotor bar induced emf, rotor bar current, stator winding induced emf and electromagnetic torque are calculated. The accuracy of the analytical model is verified by comparing its results with finite element method (FEM).

Index Terms—Analytical model, induction motor, magnetic field, air gap, finite element method.

I. INTRODUCTION

Induction motors (IM) form the major part of all electrical load globally. The recent advancement in power electronics has led the widespread use of IM in variable speed applications. The design of IM normally requires a large number of iterative computations based on the selection of different configurations. The performance of IM needs to be evaluated, to check if it is as per the desired requirements. The performance computation using finite element method (FEM) is time consuming and is not suited for iterative design procedure. Therefore, an alternative method is required to quickly compute the IM performance, which can be in the form of analytical model. The creation of an analytical model is one of the most important steps in machine design. The analytical model can be easily developed using air gap field of an IM [1], [2], [3].

Several research works have developed the analytical model of brushless permanent magnet dc (BLDC) motor [4], [5]. These works have studied the magnetic field in the air gap and effect of stator slotting on field distribution. In this paper an analytical model is developed to study the performance characteristics of cage rotor IM.

II. ANALYTICAL MODEL

The general layout of the cage rotor IM for calculating the air gap field due to a single stator current carrying coil accommodated in stator slot, which is represented as dot and cross mark is shown in Fig. 1. The important symbols and their values taken for various parameters in this paper are described in Table I. To facilitate the analysis, the permeability of stator and rotor yoke is assumed to be infinite. The Laplacian

equation for scalar magnetic potential ($\phi(\alpha, r)$) in the air gap in polar coordinate is given by (1).

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \alpha^2} = 0 \quad (1)$$

The general solution given by (2) for $\phi(\alpha, r)$ can be obtained By solving (1) with proper boundary conditions [4].

$$\phi(\alpha, r) = \sum_v (A_v r^v + B_v r^{-v}) (C_v \cos v\alpha + D_v \sin v\alpha) + (A_o \ln r + B_o) (C_o \alpha + D_o) \quad (2)$$

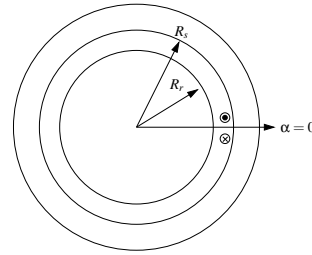


Figure 1. Schematic of a single coil representation.

Table I
 SYMBOLS AND SPECIFICATION

Parameter	Symbol	Specification
Inner radius of stator yoke	R_s	160.2 (mm)
Outer radius of rotor yoke	R_r	157.5 (mm)
Core Length of motor	L	449 (mm)
Air gap length	δ	2.7 (mm)
Instantaneous phase current	i_s	
Maximum stator phase current	i_m	25 (A)
Reference speed	N_r	3550 (rpm)
Frequency	f	50 Hz
Number of poles	P	2
Number of stator slots	Q_s	24
Number of rotor slots	Q_r	18
Coil span of a stator coil	SP	10 slots
Number of turns in a coil	N	30

Considering stator permeance function (λ_{fs}) and rotor permeance function (λ_{fr}), the radial component of the magnetic field for n number of coils derived from (2) is given by (3). Where, K_{sov} , K_{pv} and $F_v(r)$ are functions as derived in [4].

The effects of stator and rotor slots has been considered using the permeance functions (λ_{fS} and λ_{fR}) as derived in [4].

$$B_S(\alpha, r, t) = \lambda_{fS} \lambda_{fR} \sum_n \left[\frac{2\mu_o N i_S}{\pi} \frac{1}{\delta} \sum_v \frac{1}{v} K_{sov} K_{pv} F_v(r) \cos v\alpha \right] \quad (3)$$

In case of three-phase IM, carrying three-phase balanced current, formed by n number of interconnected coils, the radial component of the air gap field is obtained from (3) and is shown in Fig. 2.

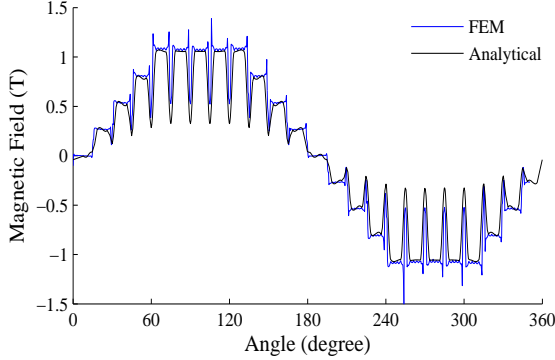


Figure 2. Stator magnetic field in the air gap.

The flux produced by stator field causes emf to induce in the rotor bars. For a rotor having Q_R rotor bars, the flux linked by a single rotor bar is given by (4).

$$\Phi_R = L \int_0^{\frac{2\pi}{Q_R}} B_S(\alpha, r, t) R_r d\alpha \quad (4)$$

Then the induced emf in the rotor bar is given by (5).

$$e_R = -\frac{d\Phi_R}{dt} \quad (5)$$

The induced rotor bar current is determined using the impedance of rotor bar (Z_b) and end ring (Z_{rg}) is given by (6).

$$i_R = \frac{e_R}{2(Z_b + Z_{rg})} \quad (6)$$

Z_b and Z_{rg} contains resistance and leakage reactance of rotor bar and end ring respectively. The rotor air gap field produced by the induced rotor bar currents is determined using the expression of field produced by a single conductor as given by (7).

$$B_R(\alpha, r, t) = \lambda_{fS} \lambda_{fR} \sum_{Q_R} \left[\frac{\mu_o i_R}{\pi} \frac{1}{\delta} \sum_v \frac{1}{v} K_{sov} F_v(r) \sin v\alpha \right] \quad (7)$$

The resultant air gap field B_{res} due to the combined effect of both stator (3) and rotor (7) current is as shown in Fig. 3.

The resultant air gap flux links the stator winding to induced emf in it. The flux linked by a single stator coil is given by (8).

$$\Phi_S = L \int_{-\frac{2\pi SP}{2Q_S}}^{\frac{2\pi SP}{2Q_S}} B_{res}(\alpha, r, t) R_s d\alpha \quad (8)$$

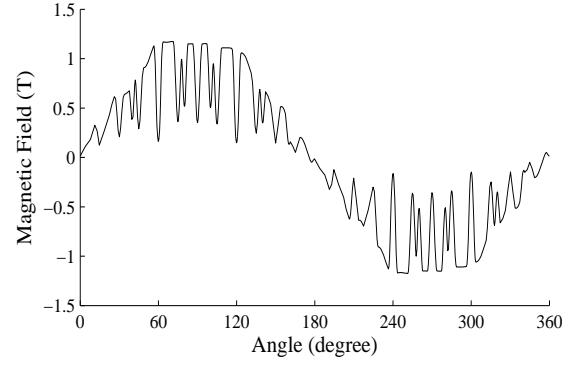


Figure 3. Resultant magnetic field in the air gap.

The induced stator winding emf per phase considering a flux leakage factor of L_f is given by (9).

$$e_S = -N \frac{d(L_f \Phi_S)}{dt} \quad (9)$$

Then the stator applied voltage per phase can be obtained and is given by (10), where $Z_S = R_S + jX_S$ is the per phase stator winding leakage impedance.

$$V_{ph} = e_S + i_S Z_S \quad (10)$$

The electromagnetic torque for the IM can be calculated using (11).

$$T_e = -\frac{P}{2} \frac{\pi \delta r L}{\mu_o} B_R B_S \sin \lambda \quad (11)$$

Where λ is the space angle between stator (B_S) and rotor (B_R) magnetic fields. Thus, from (1) to (11) complete analytical model for IM has been derived.

III. CONCLUSION

This paper presented an analytical approach to calculate performance parameters such as rotor bar induced emf, rotor bar current, stator winding induced emf and electromagnetic torque. Air gap field due to stator winding with slotting effect was validated with that obtained with FEM. Developed model of the IM provides an easy and efficient alternative usually time consuming FEM.

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