# Commutation Analysis for High-Speed Universal Motors

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*Abstract*—This paper presents a rigorous commutation analysis of high-speed universal motors that takes into account the brush-to-bar voltage drop. We used time-stepping finiteelement analysis to successfully predict the commutating currents. Tests in a 50-Hz, 100-V, 43,000-r/min universal motor validated the calculated armature current. Simulation results show the influence of the rotor speed and brush shift.

*Index Terms*—Brushes, commutation, finite element methods, universal motors.

## I. INTRODUCTION

With advantages of high starting torque, high speed and low cost, universal motors are still widely used in home appliances (e.g., vacuum cleaners and mixers) and power tools (e.g., drills and saws) [1]. Increased speed is desirable for high output power. However, it is difficult to implement the ideal commutation with respect to high-speed operation because the time cycle of the commutation becomes very short. It is almost impossible to measure the commutating coil current during high-speed operation that is greater than 40,000 r/min. A rigorous analysis to predict the commutating coil current exactly is crucial for optimal design.

Approaches for commutation analysis of universal motors are loosely classified as one of the following two methods: 1) analysis using equivalent circuits [2]-[4] and 2) numerical analysis using a finite-element method (FEM) [5]-[7].

This paper presents a rigorous commutation analysis of high-speed universal motors that takes into account the brushto-bar voltage drop. The brush-to-bar voltage drop versus the brush current density exhibits a saturation curve in the brush material used in the motors of vacuum cleaners [7]. In this paper, we approximate the saturation curve by using an exponential function. Tests in a 50-Hz, 100-V, 43,000-r/min universal motor validated the calculated armature current. Simulation results show the influence of the rotor speed and brush shift.

#### II. METHOD FOR ANALYSIS

Fig. 1 shows a cross section of an experimental motor that has two-pole series windings. If the motion of the commutation bar under the brush for a period equivalent to one slot pitch is visualized in slow motion, it is evident that the process is not continuous. The process consists instead of a sequence of switching operations caused by coils that are either beginning or completing the commutation.

In this paper, we present the fundamental equations for the magnetic field in the motor in two-dimensional rectangular coordinates as follows:



Fig. 1. Cross section of the experimental motor.



$$\frac{\partial}{\partial x} \left( v \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left( v \frac{\partial A}{\partial y} \right) = -J_0 - J_{s1} - J_{s2}$$
(1)

where A is the z component of magnetic vector potential A,  $J_0$  is the current density of the armature current  $i_0$ , and  $J_{s1}$  and  $J_{s2}$  are the current density of the short-circuited coil currents  $i_{s1}$  and  $i_{s2}$ , respectively

Fig. 2 shows the circuit of the experimental motor. The voltage and current equations of the universal motor are as follows [7]:

$$e_{f} + e_{a} + (r_{a} + r_{f})i_{0} + v_{b1} + v_{b3} + (L_{a0} + L_{f0})\frac{\partial i_{0}}{\partial t} = v_{0} \quad (2)$$

$$e_{s1} + r_{s}i_{s1} - v_{b1} + v_{b2} + L_{s0}\frac{\partial i_{s1}}{\partial t} = 0$$
(3)

$$e_{s2} + r_{s}i_{s2} - v_{b2} + v_{b3} + L_{s0}\frac{\partial i_{s2}}{\partial t} = 0$$
(4)

where  $v_0$  is the terminal voltage;  $e_f$  and  $e_a$  are the induced voltages of the field and armature windings, respectively;  $e_{s1}$  and  $e_{s2}$  are the induced voltages of the short-circuited coils;  $r_f$ ,  $r_a$  and  $r_s$  are the resistances of the field, armature and short-circuited coil windings, respectively;  $L_{f0}$ ,  $L_{a0}$  and  $L_{s0}$  are their end-winding leakage inductances, respectively;  $v_{b1} - v_{b3}$  are the brush-to-bar voltage drops; and  $i_1 - i_3$  are the currents that pass the commutator bars located under the brush.





Fig. 3 shows the brush voltage drop versus the brush current density. There are two curves. One curve is for typical characteristics of the brush material in vacuum cleaner motors [7], where the saturation voltage  $V_s$  is approximately 1.7 V. The other curve is for the characteristics of the brush material in the experimental motor, where  $V_s$  is approximately 5.0 V. The brush-to-bar voltage drop  $v_{b1} - v_{b3}$  is approximated as follows:

$$v_{bn} = V_s \left\{ 1 - \exp\left(-k \times \frac{i_n}{S_{bn}}\right) \right\}$$
(5)

where  $i_n$  is one of the currents  $i_1-i_3$  that pass the commutator bars under the brush, k is a constant that can be obtained from the measured curve, and  $S_{bn}$  is one of the contact areas of the brush-to-bar  $S_{b1}-S_{b3}$  [7].

The vector potential and currents can be obtained by using time-stepping finite element analysis [7] to solve equations (1) -(5).

### III. RESULTS

Fig. 4 shows commutating coil currents versus brush shift angle for rotor speed of 43,000 r/min and  $V_s$  of 1.7 V. The currents are shown from the initiation to completion points of commutation. Given that the brush shift angle is large, it is evident that the commutation curve changes from under commutation to over commutation. We found that the gradient of the current at the trailing edge of the brush is gentle. A sufficiently large brush angle is ideal from the view-point of the brush voltage drop (brush life). However, given that the brush shift angle is larger, the armature current is larger. Therefore, a sufficiently small brush angle is ideal from the view-point of copper loss (efficiency). From both viewpoints, an angle of 24.5° is ideal in the experimental motor.

Fig. 5 shows the calculated and measured armature currents at 43,000r/min; the agreement between the two is excellent.

## IV. CONCLUSIONS

This paper presented a rigorous commutation analysis of high-speed universal motors that takes into account the brushto-bar voltage drop. Tests in a 50-Hz, 100-V, 43,000-r/min universal motor validated the calculated armature current. Simulations showed that the brush shift, as well as the saturation voltage and rotor speed (data not shown), influences the commutating coil current and armature current.

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