Non Linear 2D Time Domain eddy current calculation for laminated iron cores

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***Abstract*— The main purpose of homogenization is to reduce computation costs. In this paper, we present a 2D time domain homogenization technique for linear and nonlinear laminated iron cores at low frequencies. This approach is based on the inclusion of eddy current losses directly in the constitutive law. It is completed by taking into account the insulating layers (fill factor). The used formulation and eddy current calculations are detailed. The developed model is validated by comparing with a 3D simulation.**

***Index Terms*—Finite element method, Eddy currents, Homogenization, Magnetic cores.**

I. Introduction

Cores in electromagnetic devices are usually laminated in order to reduce eddy current loops, and losses. However laminated cores bring new simulation constraints. For example, to reach meshing requirements according to the skin depth (2 or 3 finite elements on each skin depth thickness) can lead to prohibitive systems to solve. Moreover, the mesh quality can be deteriorated (very thin elements, difficulty to connect elements).

To fix these computation problems, different homogenization techniques are generally proposed. Among these, a method is presented in [1]-[2]. It is based on the inclusion of the eddy current losses directly in the field calculation via the constitutive law.

In this method, eddy current losses are calculated analytically using the 1D Maxwell-Faraday equation on one sheet iron [3]. The laminated core is replaced by an homogenized one with equivalent properties with a modified $h(b)$ law.

In this paper, this method is detailed and developed for linear and nonlinear materials. It is completed by taking into account the fill factor. In order to validate this approach, a 2D time domain linear and non linear laminated open core transformer are studied and performed.

II. Formulation And Eddy Current Calculations

The main aim is to define the equivalent constitutive law $h(b)$ of the homogenized core, which takes into account the eddy current losses.

Consider a linear conducting sheet with permittivity μ and conductivity σ. The 1D Maxwell-Faraday equation for one sheet iron in the case of Fig. 1. leads to:

$$\frac{\partial e_x}{\partial z} = \frac{\partial b_y}{\partial t} \qquad (1)$$

where e_x is the x-component of the electric field $\boldsymbol{e}$ (Fig. 1). The eddy current losses are then given by:

$$P_J = \int_\Omega \boldsymbol{e} \cdot \boldsymbol{j} \, d\Omega = \int_\Omega \sigma e_x^2 \, d\Omega \qquad (2)$$

where Ω is the sheet iron volume.

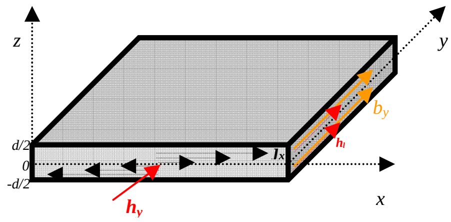

Fig. 1. Eddy currents in one sheet iron.

Neglecting skin effects, analytical solution of (1) allows to express eddy current losses (2) as:

$$P_J = \frac{\sigma d^2}{12} \left| \frac{db}{dt} \right|^2 \qquad (3)$$

where d is the sheet iron thickness and b, the average induction over the thickness. The equivalent constitutive law $h(b)$ at low frequencies, which take into account eddy current losses, is then given by [2]:

$$h(b) = [\nu(b)]b + \frac{\sigma d^2}{12} \frac{db}{dt}. \qquad (4)$$

A. Insulating layers

Let us now consider insulating layers of reluctivity ν_0 between the sheet irons. The fields in the homogenized core, denoted $\boldsymbol{h}_{//}$ and $\boldsymbol{b}_{//}$, are given by:

$$\boldsymbol{h}_{//} = \boldsymbol{h} \quad \text{and} \quad \boldsymbol{b}_{//} = \lambda \boldsymbol{b} + (1-\lambda)\boldsymbol{b}_0 \qquad (5)$$

where λ represents the fill factor of the laminated core, and $\boldsymbol{b_0}$, the induction in the air layer. After some computations, the modified constitutive law $h_{//}(b_{//})$ is defined as [2]:

$$\boldsymbol{h}_{//} = \nu_{//}(\boldsymbol{b}_{//})\boldsymbol{b}_{//} + \frac{\sigma_{//} d^2}{12} \frac{d\boldsymbol{b}_{//}}{dt} \qquad (6)$$

where $\nu_{//}$ and $\sigma_{//}$ are the homogenized reluctivity and conductivity respectively, given by:

$$\nu_{//} = \frac{1}{\lambda\mu + (1-\lambda)\mu_0} \quad \text{and} \quad \sigma_{//} = \frac{\nu_{//}}{\nu}\sigma . \qquad (7)$$

B. *Finite element formulation*

It should be noticed that, in a 2D simulation, ***b*** and ***h*** fields are tangential to the plane. However, ***e*** and ***j*** are in the normal direction to the plane and thus they do not exist in 2D model. Eddy current losses are taken into account in the equivalent $h_{//}(b_{//})$ constitutive law. The weak form of Ampère's law is then given by the classical formulation without eddy current term:

$$R_i = L\int_{\Gamma} \left[\mathbf{curl}\, w_i \cdot \mathbf{h}_{//} - w_i\, \mathbf{j}_s \cdot \mathbf{n}\right] d\Gamma = 0 \qquad (8)$$

where L is the domain depth, w_i the shape function, and, Γ, the studied domain. $\mathbf{h}_{//}$ and $\mathbf{j}_s$ are the magnetic field and the source current density respectively. Using (6), (8) leads to:

$$R_i = L\int_{\Gamma} \left[\mathbf{curl}\, w_i \cdot \left(\mathbf{h}_{s//} + \frac{\sigma d^2}{12}\frac{d\mathbf{b}_{//}}{dt}\right) - w_i\, \mathbf{j}_s \cdot \mathbf{n}\right] d\Gamma \qquad (9)$$

where $\mathbf{h}_{s//}$ represents the static behavior of the homogenized laminated core. It should be noticed that neglecting skin effects will limit the validity of the developed model to frequencies for which the skin depth in the sheet iron is greater than its half-thickness.

Equation (9) leads to a system of nonlinear algebraic equations that can be solved by means of the Newton–Raphson method. From (5) and (7), we obtain the differential reluctivity tensor:

$$\left[\frac{\partial \mathbf{h}_{//}}{\partial \mathbf{b}_{//}}\right] = \begin{bmatrix} \nu_{//} & 0 \\ 0 & \nu_{//} \end{bmatrix} + \begin{bmatrix} 2b_x^2 \dfrac{d\nu_{//}}{d\mathbf{b}_{//}^2} & 2b_x b_y \dfrac{d\nu_{//}}{d\mathbf{b}_{//}^2} \\ 2b_x b_y \dfrac{d\nu_{//}}{d\mathbf{b}_{//}^2} & 2b_y^2 \dfrac{d\nu_{//}}{d\mathbf{b}_{//}^2} \end{bmatrix} \qquad (10)$$

$$\left[\frac{\partial \nu_{//}}{\partial \mathbf{b}_{//}^2}\right] = \frac{\lambda C Q^2}{2}\left[\frac{\partial Q^2}{\partial \mathbf{b}_{//}^2} + Q^2\right] \qquad (11)$$

where $Q = \nu_{//}/\nu$, $C = 1/\|\mathbf{b}_{//}\|\, \partial\nu/\partial\mathbf{b}$, and, $\mathbf{b}_{//}^2 = b_x^2 + b_y^2$.

III. VALIDATION

In order to validate the proposed model, we consider an open core transformer. Fig. 2 shows the compared simulations. On one hand, the 3D simulation is composed of one core and one insulating layer. On the other hand, the equivalent planar circuit on which the modified $h_{//}(b_{//})$ law is implemented. Table I gives the physical properties of the sheet iron.

The 3D device is simulated at linear magneto-harmonic case at different frequencies and compared to the 2D model at the time domain and imposing a sinusoidal source current.

TABLE . I. LAMINATED CORE PHYSICAL PROPERTIES

μ_r	300
σ (Ω^{-1}.m)	2.22 x 10^6
Core thickness (mm)	0.6
Insulating layer thickness (mm)	0.1

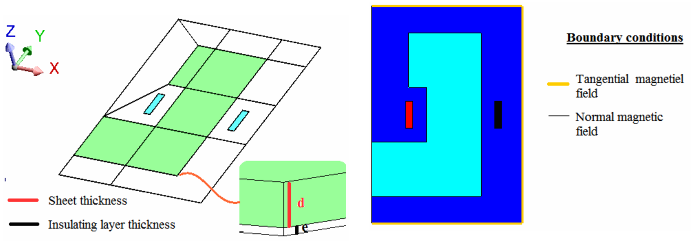

Fig. 2. 3D simulation (left) and 2D simulation (right)

Fig. 3 shows the relative error between the two simulations as a function of the frequency. As expected, the model is valid at low frequencies. For example, at 50Hz, which corresponds to a skin depth equals to 2.76mm, the error is about of 2 %.

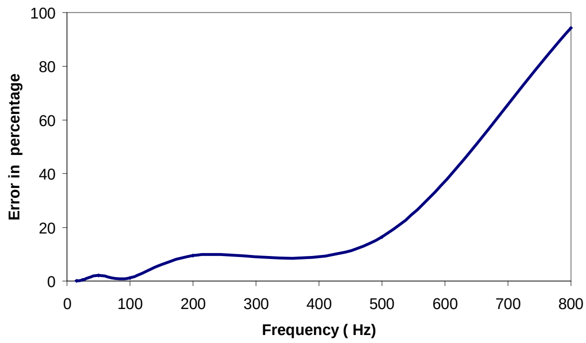

Fig. 3. Relative difference of eddy current losses as a function of the frequency

IV. CONCLUSION

In this paper, a 2D model of eddy current losses for linear material at time domain is validated. In the full paper, this model will be validated for nonlinear materials, allowing having a complete model for low frequencies. At high frequencies, a method presented in [2] and based on a polynomial orthogonal decomposition of the variation of the induction throughout the thickness of the laminations can be used.

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