

# On the Modeling of Dynamic Hysteresis Using JA and Field Separation Theories

A. P. S. Baghel, *Student Member, IEEE* and S.V. Kulkarni, *Senior Member, IEEE*

Indian Institute of Technology Bombay,  
Mumbai-400076, INDIA.  
[ajaybaghel@ee.iitb.ac.in](mailto:ajaybaghel@ee.iitb.ac.in)

**Abstract**—Magnetic materials exhibit non-linear, hysteretic and dynamic characteristics due to presence of classical eddy and anomalous losses. Existing dynamic Jiles-Atherton hysteresis models show some non-physical situations in the form of vertical lengthening of hysteresis loops. This paper proposes two inverse dynamic models based on the Jiles-Atherton approach and the field separation theory. The paper also discusses energy aspects of the JA model. The proposed models have been validated using measured curves. The model based on the field separation theory is shown to give physically correct loop shapes. It can be implemented in numerical techniques such as finite element method.

**Index Terms**— Magnetic hysteresis, Magnetic Losses, Magnetic materials

## I. INTRODUCTION

A precise modeling of core materials needs an accurate representation its magnetic characteristics. The characteristics exhibit nonlinear, hysteretic and dynamic behaviour due to eddy and anomalous losses in electrically conducting laminations [1]. Therefore, dynamic hysteresis modeling is a prerequisite for the core characterization. Among the existing hysteresis models in the literature, the Jiles-Atherton (JA) model is widely used due to ease in numerical implementation and its fewer parameters [2]. A scheme for including dynamic losses in the original JA model is given in [3]. Recently, two inverse dynamic JA models have been proposed based on the so called energy balance principle [4-5]. It is interesting to note that the two models give different expressions for the inverse dynamic JA model. In [5], one of the parameters (i.e., an eddy loss coefficient) has been obtained with a negative sign, which is a nonphysical situation. Moreover, the existing dynamic JA models can also lead to non-physical vertical lengthening of hysteresis loops when dynamic losses are included [6]. Another major concern about the JA model is that the energy function  $\int M dB_e$  used in it has been shown equivalent of co-energy in [6].

The present paper discusses two approaches for inclusion of dynamic losses. The first one based on the JA approach has been shown to give similar performance as that of the approach given in [4]. However, the model still results in nonphysical vertical lengthening of loops. The paper also clarifies the energy aspects of the JA model, which suggests that an alternative approach based on the field separation should be used for inclusion of dynamic losses. The model based on the field separation gives better loop shapes and does not lead to any nonphysical solution. Both dynamic models need seven parameters which, in this work, have been obtained

from an experimental curve using a parameter identification technique as elaborated in [7]. Simulation results are supported by experimental measurements.

## II. DYNAMIC HYSTERESIS MODELING

The static JA model is based on a magnetization process involving domain wall motion with pinning effects. The model has two components, viz. reversible magnetization  $M_{rev}$  and irreversible magnetization  $M_{irr}$  [2]. The model is defined in terms of five parameters which can be obtained from a measured curve using a hybrid identification technique. Dynamic losses can be defined as an addition of the classical eddy current and anomalous losses [8].

### A. A Dynamic hysteresis model based on the JA theory

The energy balance in the presence of the classical eddy current and anomalous losses can be written as [3],

$$\mu_0 \int M_{an}(H) dH_e = \mu_0 \int M(H) dH_e + \mu_0 k \delta (1-c) \int \left( \frac{dM_{irr}}{dH_e} \right) dH_e \quad (1)$$

$$+ k_e \int \left( \frac{dB}{dt} \right)^2 dt + k_a \int \left( \frac{dB}{dt} \right)^{3/2} dt$$

$$\frac{dM}{dB} = \frac{\delta_M (M - M_{an}) - k \delta c (dM_{an}/dH_e)}{\mu_0 [(1-\alpha)(\delta_M (M - M_{an}) - k \delta c (dM_{an}/dH_e)) - k \delta - P_d(t)]} \quad (2)$$

where,  $M$  and  $M_{an}$  are the total and anhysteretic magnetizations,  $B$  and  $H$  are the magnetic field and induction,  $\mu_0$  is the permeability of free space, and  $\delta$  is the directional parameter which depends on the rate of change of input field.  $k_e$  and  $k_a$  are eddy loss coefficient and anomalous loss coefficient. The five static model parameters, as given with their physical interpretation in [7], are  $M_s$ ,  $k$ ,  $a$ ,  $c$  and  $\alpha$ .

An inverse dynamic JA model equation (2) derived from equation (1) may lead to vertical stretching of loops due to inclusion of dynamic losses ( $P_d(t)$ ) as demonstrated in the next section.

### B. A dynamic hysteresis model based on the field separation approach

The JA energy function is equivalent to the classical energy function for a stabilized closed loop [9]. The classical function and the JA energy function are equivalent as shown in Fig. 1. Since the classical eddy losses and excess losses are independent of magnetization law  $B(H)$  and they depend on the periodic nature of magnetic induction, the loss separation can be proved equivalent to the *field separation* [10] which can be obtained using the energy balance equation as,

$$\oint_{\text{cycle}} H(B) \cdot dB = \oint_{\text{cycle}} H_{\text{hyst}}(B) \frac{dB}{dt} dt + \oint_{\text{cycle}} \frac{dW_{EC}}{dt} dt + \oint_{\text{cycle}} \frac{dW_A}{dt} dt \quad (3)$$

$$H_{\text{Total}} = H_{\text{hyst}} + H_{EC} + H_{AN} \quad (4)$$

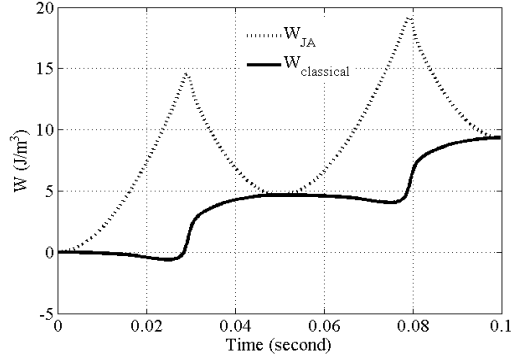


Fig. 1 The JA and classical energy functions

The hysteresis field  $H_{\text{hyst}}$  is calculated using the static JA model, which is modified on account of fields ( $H_{EC}$  and  $H_{AN}$ ) associated with the classical eddy and anomalous losses. This can be done using the modification of the effective field ( $H_e$ ) in the model.

### III. RESULTS AND DISCUSSIONS

The two presented models, based on JA approach and the field separation approach (FSA), are validated using measured curves. Dynamic model parameters are obtained by optimizing the measured curve at 50 Hz, which are subsequently used for computation of curves at other frequencies.

Table- I

Symbols	Optimized parameters
$M_s(A/m)$	$1.24 \times 10^6$
$a(A/m)$	10
$k(A/m)$	25
$\alpha$	$3.3 \times 10^{-5}$
$c$	0.30
$k_e(m/\Omega)$	$2.52 \times 10^{-2}$
$k_a(A/\Omega)^{1/2}$	$1.15 \times 10^{-2}$

Hysteresis loops computed using the two models are shown in Figs. 2 and 3.

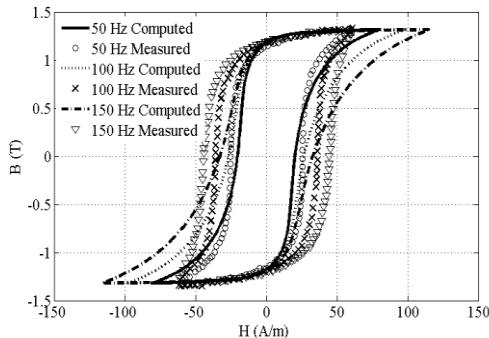


Fig. 2 Computed dynamic loops (Model-I-JA)

The results obtained from the JA theory based model show higher errors due to nonphysical vertical lengthening as evident from Fig. 2. Loops, computed using FSA, are accurate with no unrealistic features (Fig. 3).

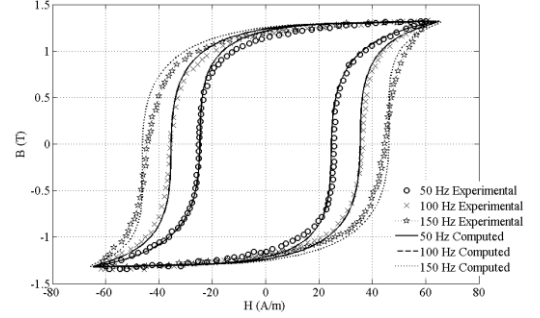


Fig. 3 Computed dynamic loops (Model-II-FSA)

### IV. CONCLUSIONS

The paper has presented two inverse dynamic models, one is based on the original JA approach and the second is developed using the field separation theory. The JA energy function has been shown to be equivalent to the classical energy function for a stabilized closed loop. This equivalence provides a correspondence between the loss separation and field separation approaches. The proposed models have been validated using experimental curves. The model based on the field separation theory is shown to give physically correct loop shapes with reasonable accuracy levels. The model can be useful for determination of core losses accurately.

### REFERENCES

- [1] S. E. Zirka, Y. I. Moroz, P. Marketos, A. J. Moses, D. C. Jiles, and T. Matsuo, "Generalization of the Classical Method for Calculating Dynamic Hysteresis Loops in Grain-Oriented Electrical Steels," *IEEE Transactions on Magnetics*, Vol. 44, pp. 2113-2126, 2008.
- [2] D. C. Jiles, J. B. Thielke, and M. K. Devine, "Numerical determination of hysteresis parameters for the modeling of magnetic properties using the theory of ferromagnetic hysteresis" *IEEE Transactions on magnetics*, Vol. 28, pp. 27-35, 1992.
- [3] D. C. Jiles, "Frequency Dependence of Hysteresis Curves in Conducting Magnetic Materials," *Journal of Applied Physics*, Vol. 76, pp. 5849-5855, 1994.
- [4] K. Chwastek, "Modeling of Dynamic Hysteresis Loops using the Jiles-Atherton approach," *Mathematical and Computer Modeling of Dynamical Systems*, Vol. 15, pp. 95-105, 2009.
- [5] H. Li, Q. Li, X. Xu, T. Lu, J. Zhang, and L. Li, "A modified Method for Jiles-Atherton Hysteresis Model and Its Application in Numerical Simulation of Devices Involving Magnetic Materials," *IEEE Transactions on Magnetics*, Vol. 47, pp. 1094-1097, 2011.
- [6] S. E. Zirka, Y. I. Moroz, R. G. Harrison, and K. Chwastek, "On physical aspects of the Jiles-Atherton hysteresis models," *Journal of Applied Physics*, Vol. 112, pp. 043916-1-043916-7, 2012.
- [7] A. P. S. Baghel and S. V. Kulkarni, "Parameter Identification of the Jiles-Atherton Hysteresis Model using a Hybrid Technique," *IET-Electric Power Application*, Vol. 6, pp. 689-695, 2012.
- [8] G. Bertotti, "Space-Time Correlation Properties of the Magnetization Process and Eddy Current Losses: Theory," *Journal of Applied Physics*, Vol. 54, pp. 5293-5305, 1983.
- [9] R. Venkataraman and P. Krishnaprasad, "Qualitative Analysis of a bulk ferromagnetic hysteresis model," *Proceedings of the IEEE Conference on Decision and Control*, Vol. 3, pp. 2443-2448, 1998.
- [10] G. Bertotti, *Hysteresis in Magnetism*, San Diego, CA: Academic, 1998.