

# A Complex-Valued Rotating Magnetic Property Model and Its Application to Iron Core Loss Calculation

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**Abstract**—This paper presents an improved complex-valued reluctivity model considering 2D magnetic properties of electrical steel sheet from the viewpoint of saving computation time and guaranteeing the solution accuracy. In this model, the effective magnetic reluctivity coefficients are calculated according to the average magnetic energy density in one time period, and the effective magnetic hysteresis coefficients computed from the magnetic hysteresis loss. To improve the modeling accuracy, the effects of higher harmonic terms besides the fundamental one of the magnetic field intensity waveform on the magnetic property are considered into this model. In comparison with the conventional E&S model, the effectiveness of the proposed model is verified with experimental results.

**Index Terms**— Core loss, Magnetic hysteresis, Rotating magnetic property, Finite element methods.

## I. INTRODUCTION

As one of highly accurate methods to simulate 2D vector magnetic properties of electric steel sheet, the E&S model has attracted more and more attentions. In this model, the magnetic anisotropy of electrical steel sheet under the alternating and the rotating magnetic flux conditions is calculated from plenty of experiment data measured from a 2D magnetic property measurement system [1-4]. Very recently, in order to save the computation time of finite element analysis considering E&S model, a complex E&S model is proposed with assumption that both magnetic flux density and magnetic field intensity vary sinusoidally with time [5]. It has mentioned in [5] that this model can only be used in low magnetic flux region because the magnetic field intensity will be gradually distorted from the purely sinusoidal waveform with the increase of magnetic flux density.

In this paper, an improved complex-valued reluctivity model is developed, in which a novel definition of the

effective magnetic reluctivity and hysteresis coefficients are proposed by considering the distorted waveform of magnetic field intensity at high magnetic flux density region. By comparing with the conventional E&S model and experimental results, it has demonstrated that the proposed model is time-saving and accurate enough for engineering application.

## II. METHOD DESCRIPTION

### A. Improved Complex Reluctivity Model

In order to describe 2D magnetic properties, the E&S reluctivity model based on the Chua-type model can be expressed as follows[1]:

$$H_k = v_{kr} B_k + v_{ki} \frac{\partial B_k}{\partial t} \quad (k = x, y) \quad (1)$$

where  $v_{kr}$  and  $v_{ki}$  are magnetic reluctivity and hysteresis coefficients, respectively,  $B$  is magnetic flux density, and  $H$  is magnetic field intensity. In this model,  $v_{kr}$  and  $v_{ki}$  are expressed as the function of time and the locus of magnetic flux density  $B$  in one time period, and the corresponding finite element analysis is transient. In order to save FEA computation time, the complex reluctivity model is expressed as follows [5]:

$$\dot{H}_k = \bar{v}_{kr} \dot{B}_k + j\omega \bar{v}_{ki} \dot{B}_k \quad (2)$$

where  $\bar{v}_{kr}$  and  $\bar{v}_{ki}$  are defined as the effective magnetic reluctivity and effective magnetic hysteresis coefficients, respectively, and calculated from the fundamental terms of measured  $B$ - and  $H$ -waveforms with 2D magnetic property measurement device.

By using a self-developed 2D magnetic properties measurement device of electric steel sheet in Fig.1(a), the measured magnetic field intensity waveforms are shown in Fig.1(b) when the controlled elliptical locus of magnetic flux density vary from 0.4T to 1.0T. From this figure, it can be seen that the  $H$ -waveform is distorted and includes high order harmonic terms when the  $B$  locus is 1.0T. Therefore, in order to improve the accuracy of complex model in (2), it is necessary that the definition of the effective magnetic reluctivity and hysteresis coefficients needs to take the high order harmonic terms of measured  $H$ -waveforms into account. In this paper, an improved complex-valued reluctivity model is proposed, and the computation of  $\bar{v}_{kr}$  and  $\bar{v}_{ki}$  are derived according to the average magnetic energy density and the magnetic hysteresis loss in one time period, respectively.

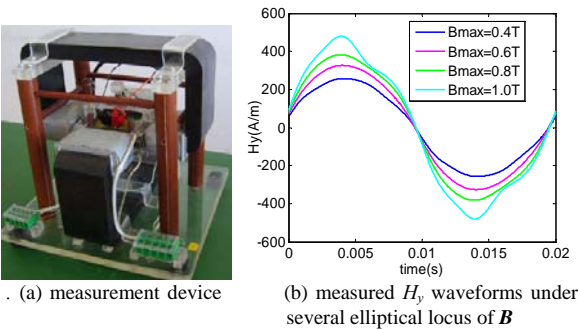


Fig.1 2-D magnetic property measurement of electrical steel sheet.

Firstly, the effective magnetic hysteresis coefficient  $\bar{v}_{ki}$  is related to hysteresis loss of electric steel sheet. The total iron loss including both alternating loss and rotational loss can be calculated as follows:

$$P_{loss} = \frac{1}{\rho T} \int_0^T \left( H_x \frac{dB_x}{dt} + H_y \frac{dB_y}{dt} \right) dt \quad \text{W/kg} \quad (3)$$

By utilizing the measurement data of magnetic field intensity corresponding to different elliptical locus of magnetic flux density, the iron loss in (3) of the electric steel sheet can be measured. Obviously, the measured loss results include the high harmonic terms of  $H$ -waveform. On the other hand, during the course of measurement, the controlled  $B$ -waveform can be expressed as follows:

$$B_k = B_{mk} \sin(\omega t + \varphi_k) \quad (k = x, y) \quad (4)$$

where  $B_{mk}$  is the amplitude of  $B_k$ . According to E&S model, the corresponding  $H$ -waveform can be deduced from (4) as follows:

$$H_k = \bar{v}_{kr} B_{mk} \sin(\omega t + \varphi_k) + \bar{v}_{ki} B_{mk} \omega \cos(\omega t + \varphi_k) \quad (5)$$

Substituting (4) and (5) into (3), the calculated iron loss can be obtained. By considering the measured and calculated iron loss simultaneously, the effective magnetic hysteresis coefficients  $\bar{v}_{ki}$  can be derived as follows:

$$\bar{v}_{ki} = \frac{2}{B_{mk}^2 \omega^2 T} \int_0^T H_k(t) \cdot \frac{dB_k(t)}{dt} dt \quad (6)$$

where  $H_k(t)$ ,  $B_k(t)$  and  $B_{mk}$  are obtained from the 2D magnetic property measurement data. For a definite  $B$  locus described with three elliptical parameters: the long axis length  $B_{max}$ , the inclination angle  $\theta$  and the axis ratios  $\alpha$ , the effective magnetic hysteresis coefficients  $\bar{v}_{ki}$  are constant.

Secondly, the effective magnetic reluctivity coefficients  $\bar{v}_{kr}$  are related to the average magnetic energy density in one time period. The average magnetic energy density can be calculated as follows:

$$W_e = \frac{1}{T} \int_0^T \frac{1}{2} H \cdot B dt \quad (7)$$

By applying the similar derivation course as that for the  $\bar{v}_{ki}$  and taking account of both the measured energy density and the calculated one, the effective reluctivity coefficients  $\bar{v}_{kr}$  can be calculated as follows:

$$\bar{v}_{kr} = \frac{2}{B_{mk}^2 T} \int_0^T H_k(t) \cdot B_k(t) dt \quad (8)$$

Similarly, the effective magnetic reluctivity coefficients  $\bar{v}_{kr}$  are also the function of  $B_{max}$ ,  $\alpha$  and  $\theta$ .

In a word, the coefficients  $\bar{v}_{ki}$  and  $\bar{v}_{kr}$  in the complex-valued reluctivity model proposed in this paper are calculated

from the viewpoint of measured iron loss and energy density in order to take the high order harmonic terms of measured  $H$ -waveforms into account. The detailed discussion for  $\bar{v}_{ki}$  and  $\bar{v}_{kr}$  will be given in the extended paper.

### B. Governing equation

The governing equation under consideration of the proposed complex reluctivity model can be expressed as follows:

$$\frac{\partial}{\partial x} \left( \bar{v}_{yr} \frac{\partial \dot{A}}{\partial x} + j\omega \bar{v}_{yi} \frac{\partial \dot{A}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \bar{v}_{xr} \frac{\partial \dot{A}}{\partial y} + j\omega \bar{v}_{xi} \frac{\partial \dot{A}}{\partial y} \right) = -\dot{J} \quad (9)$$

where  $\dot{A}$  is complex magnetic vector potential, and  $\dot{J}$  is the complex exciting current density. With the proposed complex-valued reluctivity model, the efficiency of finite element analysis can be improved.

## III. RESULTS

A three-phase transformer core model is shown in Fig.2, and Fig.3 shows the iron loss distribution. Table I lists the compared results. More results and discussions will be given in the extended paper.

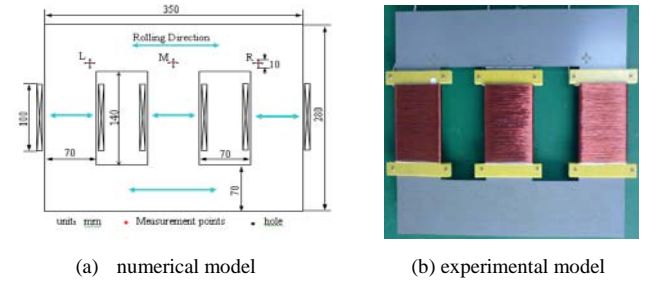


Fig.2 Three-phase transformer core model.

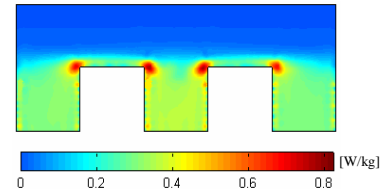


Fig.3 Iron loss distribution.

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TABLE I

COMPARISON OF COMPUTATION RESULT OF IRON LOSS UNDER DIFFERENT EXCITING VOLTAGE (UNIT: mW)

Exciting voltage	1V	1.5V	2V	CPU time/s
Improved E&S model [4]	215.65	298.13	405.89	8856
Proposed model	218.79	297.96	402.8	570
Experimental results	240.9	316.0	428.9	