Homogenized Magnetostatic Analysis of Periodic Structure with Anisotropy

Y. Ito¹, H. Igarashi¹, *Member*, *IEEE*

¹Graduate School of Information Science and Technology, Hokkaido University, Kita 14, Nishi 9, Kita-ku, 060-0814, Sapporo, Japan

yasuito@em-si.eng.hokudai.ac.jp

Abstract—This paper presents a novel numerical method to compute the electromagnetic properties of soft magnetic composite (SMC) with anisotropy. It is assumed that SMC is composed of homogeneous electromagnetic structure with periodicity. In the present method, a unit domain which consists of arbitrary-shaped magnetic particle coated by non-magnetic material is analyzed by applying the boundary condition which is computed from superposition of magnetic fields generated by the magnetized particles. The magnetization of the particles is computed under the assumed boundary condition. The boundary condition and magnetization are determined through iteration of the above procedure. The present method can be extended to the analysis of general periodic structure with anisotropy.

Index Terms—Soft magnetic composite, Periodic structures, Finite element method

I. INTRODUCTION

Composite materials such as soft magnetic composite (SMC) and epoxy resin composed of carbon fibers are widely used because they have many advantages: lower eddy current losses in SMC [1]-[2] and lightweight of epoxy resin [3]. Thus it is important to develop numerical methods in order to analyze their microscopic and macroscopic properties. They are often assumed to be composed of homogeneous structures with periodicity for simplicity. The macroscopic property of SMC can be analyzed by the homogenized method if the magnetic field in the unit cell has right-left symmetry so that the boundary condition can be appropriately imposed [4]-[5]. However, it has been difficult to analyze the anisotropic properties of the periodic structure. This is due to the fact that the boundary condition for the unit cell is unknown when external field is imposed in oblique direction and/or particles in the unit cell have arbitrary shape. The isotropic properties could be computed by analyzing the field in a large domain which includes sufficiently many unit cells. However, this brute-force method would need prohibitive computational cost.

This paper presents a novel numerical method to compute the anisotropic electromagnetic properties of SMC composed of periodic unit cell. In the present method, the vector potential on the boundary of the unit cell is computed from superimposition of magnetic fields generated by the magnetized particles. The magnetization of the particles is analyzed under the assumed boundary condition for the unit cell. This procedure is repeated until convergence. Consequently, the boundary condition as well as magnetization are determined by the present method so that the anisotropic magnetic properties are obtained.



Fig. 1. Outline of the present method

II. PRESENT METHOD

Let us consider SMC which consists of homogeneous magnetic particles of arbitrary shape which are coated by non-magnetic material as shown in Fig. 1. We consider the magnetic particles of the same size for simplicity. The magnetic field without electric currents to which the uniform magnetic induction B_0 is imposed is governed by

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A}\right) = 0 \tag{1}$$

where μ , A are the permeability and vector potential.

In the first step of the present method, the unit domain which consists of a magnetic particle is assumed to be located in sufficiently large air region for which we set the Dirichlet boundary condition, as shown in Fig. 1, which is called model 1. By analyzing the magnetostatic field using the finite element (FE) method in this region, the magnetization M(r) is calculated by

$$\boldsymbol{M}(\boldsymbol{r}) = \frac{\boldsymbol{B}}{\mu_0} - \boldsymbol{H} = \frac{\nabla \times \boldsymbol{A}}{\mu_0} - \frac{\nabla \times \boldsymbol{A}}{\mu}$$
(2)

where r, B, H and μ_0 are the positional vector of M, magnetic flux density, magnetic field and permeability of vacuum. When assuming that SMC is composed of the periodic unit cells, all the magnetization in the unit is identical, that is, $M^0 = M^1 = \cdots = M^n$ where n denotes the index which identifies the unit cells surrounding the unit cell with n=0 to which we pay attention. Thus the vector potential $A_i^b(r')$ at the point $P_i(r')$ on the boundary of unit domain can be calculated by the Biot-Savart law as follows:



Fig. 2. Flowchart of the present method

$$A_{i}^{b}(\mathbf{r}') \approx \sum_{l=0}^{n} \sum_{k=0}^{e} \frac{\mu_{0}}{4\pi} \frac{M_{k}^{l} \Delta v_{k}^{l} \times \mathbf{r}_{k}''}{|\mathbf{r}_{k}''|^{3}}$$
(3)

where i, e, Δv and \mathbf{r}' are the index of P, number of FE, volume of FE, positional vector of \mathbf{A}^{b} and $\mathbf{r}_{k}''^{1} = \mathbf{r}_{i}' - \mathbf{r}_{k}^{1}$. Obtaining the initial magnetization in model 1, \mathbf{A}^{b} is computed from (3). Then (1) is solved under the Dirichlet boundary condition with \mathbf{A}^{b} , \mathbf{M} is computed from (2). This iterative procedure is shown in Fig. 2.

III. NUMERICAL RESULTS AND CONCLUSIONS

To test the validity of the present method, a simple model shown in Fig. 3 is analyzed. In this model, the unit cell which includes a cubic material and air region is immersed in the external magnetic field B_0 parallel to *z*-axis. Because there is a right-left symmetry in this model, the boundary condition is a priori known: we impose Neumann and Dirichlet conditions for A^b on the top-bottom and right-left boundaries, respectively. This model can be analyzed the conventional FEM. The magnetic flux density obtained by the present method and the conventional FEM is shown in Fig. 4. We can see in Fig. 4 that they are almost identical. We also find that the maximum values in the magnetic flux density obtained by both methods are in good agreement.

In the full paper, the anisotropic magnetic properties obtained by the present method will be shown in detail. Moreover, the convergence and accuracy of the present method will also be discussed.

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Fig. 3. Analysis Models



Fig. 4. Distribution of magnetic flux density on x-z plane for model 2