Modelling of Several Concentric Layers of Superconducting Filaments

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***Abstract*—The knowledge of the critical current density in the superconducting filaments is an important issue of the LHC main magnets design during the construction at CERN. A new method for modelling a superconducting strand which consists of several concentric layers of the filaments is proposed in this paper. The superposition theorem in the circuit theory is applied in this method. The numerical results obtained by using the finite element method demonstrate the coupling and decoupling behaviours between the superconducting filaments via the resistive matrix. The comparison with the earlier work on the superconductor modelling are presented and discussed.**

***Index Terms*—Electromagnetic coupling, finite element methods, magnetization, superconducting filaments**

I. Introduction

In recent years, the superconducting multifilamentary strands composing the Rutherford cables of the LHC main magnets are produced at CERN [1]. In order to design these magnets, the knowledge of the current density distribution in the filaments is necessary [2]. For several years, the coupling and decoupling behaviours between the superconducting filaments via the resistive matrix can be described by the numerical results obtained by using the finite element method.

A strand is normally made up of several concentric layers of the filaments. The aim of this work is to model a strand which consists of several layers of the superconducting filaments. For that, we propose a new method by using the superposition theorem and Ohm's law from the circuit theory. In this paper, we demonstrate the coupling and decoupling between the superconducting filaments in an applied field. The current density distributions and the magnetization hysteresis loops are presented. The comparison with the earlier work in [3] and [4] is shown too.

II. Problem Analysis

Let us consider a model of a strand composed of several concentric layers of the superconducting filaments with a finite length L, as shown in Fig. 1 (left). The filaments are arranged, within each of the layers, substantially on a circle. The innermost layer (1st layer) and outermost layer (nth layer) are made of 6 filaments and 6×n filaments respectively with one filament at the centre of the strand. For a test model in Fig. 1 (right), a strand formed of two adjacent layers of the filaments is proposed. All filaments are embedded in a normal resistive matrix. The external field is applied in the direction perpendicular to the filament axis (z axis). The current density is assumed to depend on time (t) and 2-D Cartesian coordinates (x, y). For simplicity and due to the source field distribution, we suppose that the voltages and the currents are in the form of a sinusoid. For the ith layer, we have

$$[V_{ik} \quad I_{ik}]^t = [V_i \quad I_i]^t \times sin[(k-1)2\pi/n_i] \tag{1}$$

where k = 1, 2, ... , n_i and n_i = 6×i.

Starting from the superposition theorem, by feeding only the ith layer with the voltage V_i , the AC losses P_i in the resistive matrix is obtained by

$$P_i = Y_i V_i^2 \sum_{k=1}^{n_i} sin^2[(k-1)2\pi/n_i] \tag{2}$$

where $Y_i = I_i/V_i$. Then, feeding two adjacent layers together with the voltages V_i and V_j , the AC losses P_{ij} which is equal to P_{ji} can be obtained by

$$P_{ij} = \left(Y_i V_i^2 + Y_{ij} V_i V_j\right) \sum_{k=1}^{n_i} sin^2[(k-1)2\pi/n_i] + \left(Y_{ij} V_i V_j + Y_j V_j^2\right) \sum_{k=1}^{n_j} sin^2\left[(k-1)2\pi/n_j\right]. \tag{3}$$

Furthermore, the AC losses P_i and P_{ij} can be calculated by a 2-D formulation of a harmonic problem with the imposed values $V_i = V_j = 1$ [4]. Therefore, we can deduce Y_i and Y_{ij} from (2) and (3). For a test case of two layers, we obtain

$$Y_1 = \frac{P_1}{3V_1^2} \quad , \quad Y_2 = \frac{P_2}{6V_2^2} \quad , \quad Y_{12} = \frac{P_{12} - 3Y_1V_1^2 - 6Y_2V_2^2}{9V_1V_2}. \tag{4}$$

Note that for n layers strand, the total number of computations to obtain Y is n×(n+1)/2, the total number of filaments in the strand is N+1 where

$$N = 6 \times \sum_{i=1}^{n} i. \tag{5}$$

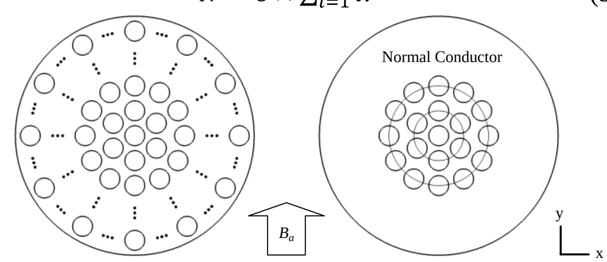

Fig. 1. Model of a strand composed of n concentric layers of the filaments.

By using Ohm's law, we obtain the relation between the currents inside the filaments and the voltages which can be written in the matrix form as follows

$$I = \begin{bmatrix} I_{1k} \\ I_{2k} \\ \vdots \\ I_{nk} \end{bmatrix} = C \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = C[Y] \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = C[Y]V \tag{6}$$

$$C = \begin{bmatrix} C_{1k} & 0 & \cdots & 0 \\ 0 & C_{2k} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{nk} \end{bmatrix} \quad , \quad [Y] = \begin{bmatrix} Y_1 & Y_{12} & \cdots & Y_{1n} \\ Y_{12} & Y_2 & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{1n} & Y_{2n} & \cdots & Y_n \end{bmatrix} \tag{7}$$

where C_{ik} are the values of the sinusoid. For the filament at the centre, due to the symmetry V_0 = 0 and naturally I_0 = 0.

Remark that the square matrix [Y] is a symmetric matrix. The dimensions of C and [Y] are N×n and n×n respectively.

III. Numerical Modelling

In order to characterize the nonlinear electric property of the superconductors, the behaviour laws between the current density and the electric field are proposed in [5] and [6]. For that, in this work, we use an extension of Bean's critical state model (see in [7]). A geometric model of the problem as in Fig. 1 is recently created by using a finite element mesh generator called Gmsh [8]. The results of the problem which can be obtained by modifying the finite element program of LGEP are the electric field. The currents circulating in the filaments at each iteration p can be derived from [3]

$$I^p = \left([A_{ev}]^t E^p + [A_v] E_0^p\right) \times \Delta t + I^{p-1} \quad (8)$$

where [A_{ev}] and [A_v] indicate the matrix of rigidity and the matrix of the electric resistance respectively [4]. E is the vector of the electric field and the voltage per unit of length in the filament E_0 is defined as follows

$$E_0 = -CV/L. \quad (9)$$

By replacing (6) in (8) and using (9), we have

$$V^p = [A]^{-1} B \quad (10)$$

where (in the case of n layers, the dimensions of [A], B and D are n×n, n×1 and N×n respectively)

$$[A] = D^t D \quad , \quad B = D^t([A_{ev}]^t E^p \times \Delta t + I^{p-1}) \quad (11)$$

$$D = C[Y] + [A_v] C \times \Delta t / L. \quad (12)$$

By using V, we obtain E_0 and then I from (9) and (8) respectively. Finally, we can solve the problem with the following matrix system

$$[M]\partial_t J + [A_e] E + [A_{ev}] E_0 = F \quad (13)$$

where [M] and [A_e] indicate the matrix of mass and the matrix of rigidity respectively. J and F are the vector of the current density and the vector of source terms respectively.

Otherwise, by changing the variables, we have

$$V = -[C^t C]^{-1} C^t E_0 \times L. \quad (14)$$

By replacing (14) in (6) and then in (8), so we have

$$E_0^p = [A]^{-1} B \quad (15)$$

where (in this case, the dimensions of [A] and B are N×N and N×1 respectively)

$$[A] = -C[Y][C^t C]^{-1} C^t \times L - [A_v] \times \Delta t \quad (16)$$

$$B = [A_{ev}]^t E^p \times \Delta t + I^{p-1}. \quad (17)$$

By using E_0 , I is obtained from (8) and then the problem can be solved with (13).

IV. Simulation Results

In order to test our model and approach to the real structure of the LHC strand, we make simulations of a strand composed of two layers of 19 superconducting filaments with a filament diameter of 7 μm in a copper matrix with $\sigma = 10^{10}$ S/m and J_c = 2,000 A/mm^2 at $B_{a,max}$ = 20 mT, 50 Hz [1].

Figure 2 shows the simulation results of the current density distributions in the modelled domain (Fig. 1 (right)) at partial (left) and total (right) penetrations. The figures on the top and the bottom show the cases of full coupling and full decoupling respectively. We can see the persistent current shells in the filaments. We observe that these results are in agreement with those in [3] and [4]. We find again that the situation of partial coupling appears when the length of the filament is a few μm. These results confirm that our model works well. In addition, we can present the current distribution in the resistive matrix and also the magnetization hysteresis loops. Moreover, our model allows us to calculate the magnetization per unit of superconductor volume versus the number of layers too.

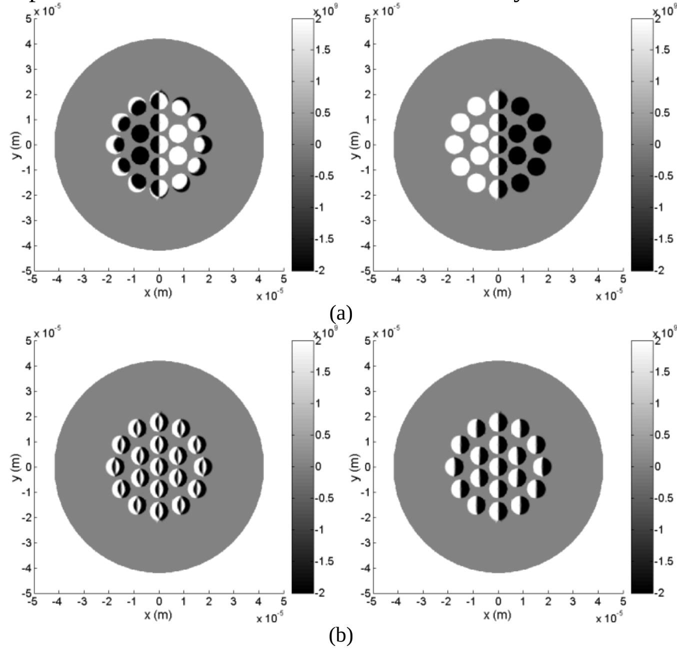

Fig. 2. Current density distributions in the superconducting filaments for a test model: (a) full coupling case and (b) full decoupling case.

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