Comparison Between Different Approaches in Homogenization : Mean-field Approach vs Full-field Approaches

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***Abstract*—This paper presents the comparison between different approaches to determine the effective properties of composite materials. Homogenization is a very useful tool when the size of heterogeneities is much smaller than the size of the device to model, because the size of the numerical model would become too big. The models presented here are Finite Element Model, FFT model and homogenization based on inclusion problems. The application studied here show similar results on a fiber-matrix composite but each method presents advantages and disadvantages which are listed in this paper.**

***Index Terms*—Homogenization, Finite Element Model, Fast Fourier Transform, Effective Properties, Multiphysics.**

I. Introduction

Composite materials are more and more used in industry because of their useful properties. Indeed, mixing different materials in a proper way enables to design a material taking advantage of the different constituents. A classical composite material an epoxy resin embedding carbon fibers, which is a good candidate to replace aluminum in shielding enclosures. The carbon fibers have on important role on the macroscopic electric conductivity whereas the epoxy resin makes the composite have good mechanical properties for enclosures as well as a light weight, which is of interest for aeronautic or automotive industry.

A complete numerical model to study a device and carrying the heterogeneous nature of the composite is impossible most of the time because of the computational cost it involves. That is the reason why homogenization approaches enable to determine the macroscopic behavior of a composite material and consider it as homogeneous in the numerical model of the device[1]. Two main classes exist in homogenization, mean-field approaches versus full-field approaches. The first one only describes the microstructure of the composite from statistical information (volume fractions, . . .) and only pieces of information about the fields can be determined, such as the mean fields per phase for example[2]. Full-field approaches need a complete description of the microstructure but can then provide the complete description of the field distribution[3], [4].

In a first part, the different approaches will be briefly presented. In a second part, the results obtained on a periodic fiber-matrix composite are shown and compared. Then in a final part, the comparison between the methods is performed, pointing out the advantages and disadvantages of each method.

II. Homogenization approaches

A. Principle

The principle of homogenization is to determine the effective properties of an heterogeneous material, seen as homogeneous in a higher scale.

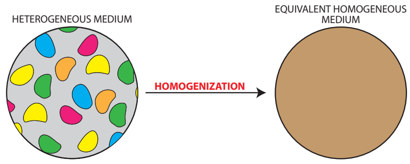

Figure 1: Principle of homogenization.

B. Full-field approach

1) Finite Element Method: The Finite Element Method is a common approach to study composite materials. The microstructure of the composite is fully described through the geometry and the mesh. Periodic conditions can be applied on a basic cell to study periodic materials. Nonlinear behavior can also be taken into account through iterative solvers, as well as coupled behavior[3].

The determination of the effective properties can be done with the computation of the macroscopic response (average over the cell) for different macroscopic loadings (imposed through the periodic conditions for example).

2) Fast Fourier Transform: The FFT modeling is an efficient approach to study periodic composites only [4]. It is based on the Maxwell equations in the Fourier space which leads to simplifications in the equations. The microstructure of the composite is fully described through a pixel image of the basic cell. The periodic nature of the fields in implicit in this approach. The algorithm is iterative and rely on a convergence parameter (like iterative solvers in FEM). Coupled behavior can also been taken into account[5].

The determination of the effective properties can be done in a similar way than for FEM.

C. Mean-field approach: based on inclusion problems

One mean-field homogenization approach is based on inclusion problems[2]. The modeling of a composite material made of n phases can be processed through n basic inclusion problems. Unlike full-field approaches, only partial information about the microstructure is needed. In the model based on inclusion problems, volume fractions of the phases and a statistical information on the distribution are required. With these pieces of information, the determination of the effective properties can be performed analytically. Nonlinear behavior can be taken into account through a linearization process. Multiphysic behavior can also be taken into account through the decomposition of the fields into different sources[6].

III. Results on a fiber-matrix material

The magnetic behavior is studied here. The studied composites are periodic fiber-matrix materials. Random microstructures (random location of the center of fibers) are created for the full-field approaches (see Fig. 2). The results in mean-field approach come from Hashin-Shtrikman estimate which is well adapted for fiber-matrix composites. The relative permeabilities of the phases are: matrix $\mu_1 = 1$ and fibers $\mu_2 = 10$.

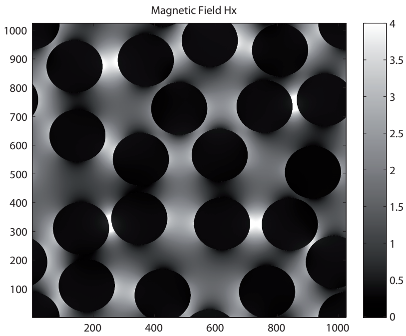

Figure 2: Example of FFT solution (1024x1024 pixels) on a random periodic microstructure for fiber-matrix composite. The solution is the magnetic field along x (Applied magnetic field along x).

Different volume fractions for the fibers are considered. Several realizations of the random microstructure are performed for each volume fraction. FEM and FFT results are very similar for the effective properties in this case (Fig. 3). Some discrepancies can be shown locally and will be presented in the full paper.

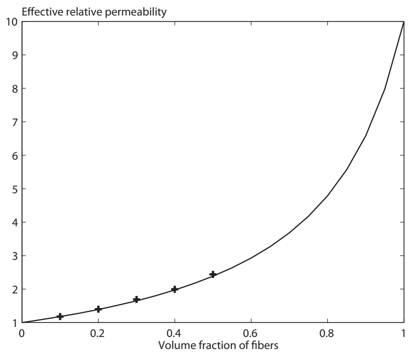

Figure 3: Effective relative permeability as a function of the volume fraction of fibers. Line: mean-field approach (Hashin-Shtrikman estimate). Crosses: FEM/FFT results.

IV. Comparison between these approaches

It can be concluded in this example that each method provides similar results for the effective properties. Nevertheless, all these models present major differences listed in Table I.

Since the mean-field approach can be performed analytically, the computation time is almost zero. The FFT model takes a few seconds for a discretization of 1024x1024 pixels. The FEM takes around 1 minute to solve on a fine mesh (but with a smaller number of unknowns than the FFT model).

Table I: Advantages and disadvantages

Method	FEM	FFT	Inclusion problems
Computation time	−	+	++
Pre-processing	−	++	+
High Frequency	++	impossible	-
Nonlinear behavior	++	++	-

Considering the pre-processing time, the FFT model is the more efficient since it only needs a pixel image of the microstructure. The geometry needs to be acquired in the FEM, which can be done thanks to image processing for example, and the mesh of the geometry needs to be controlled.

The ability of the models to work with high frequency shows that it cannot be performed with FFT since the field is implicitly periodic on the cell. FEM formulations are able to take into account higher frequencies. Some adjustments have been recently proposed for mean-field approach to take into account higher frequencies[7].

Finally, when nonlinear behavior is encountered, FEM and FFT models can provide accurate results whereas the mean-field approach can only give a coarse estimate. In fact, the field distribution is essential when dealing with nonlinear behavior and more information than mean fields per phase are necessary to obtain a suitable estimate. A first improvement is obtained through the determination of second moment of the fields which is possible in this model[8].

V. Conclusion

The comparison between different homogenization models is presented here. It shows that for linear behavior, each method give similar results for homogenization. Computation time can be a criterion to chose a model but other methods present other advantages compared to mean-field approach. The advantages and disadvantages of each method are given and more explanations will be given in the full paper. Nonlinear behavior will also be studied.

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