Adaptive Mesh Morphing Method for Numerical Analysis of Electromagneto-mechanical Coupling using Lagrangian Approach

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***Abstract*—Electromagneto-mechanical coupling effect associated with vibration induced eddy currents plays an important role in the evaluation and design of many magnetic devices, such as a tokamak. In this paper, the numerical method dealing with coupling effect based on the Maxwell's equations in Lagrangian description is upgraded by introducing an adaptive mesh morphing method to improve its simulation efficiency. Mesh morphing and finite element coefficient matrices reforming are only conducted in the time step when vibration induced eddy current becomes significant. A corresponding numerical code was developed on ANSYS platform and validated by simulating the Team 16 benchmark problem.**

***Index Terms*—Electromagneto-mechanical coupling, eddy current, Lagrangian approach, adaptive mesh morphing.**

I. INTRODUCTION

Electromagneto-mechanical coupling effect is an important factor for eddy current and structural vibration problem of some large magnetic devices, such as the dynamic behaviors of the structure of a tokamak type fusion reactor during plasma disruption. To deal with coupling effect, velocity term $\boldsymbol{v}\times\boldsymbol{B}$ was introduced to take into account the interaction between the electromagnetic (EM) fields and the structural vibration [1, 2]. There are also many papers proposing methods for numerical simulation of eddy current problems with moving bodies[3, 4]. Though limited to the problems of rigid body motion, these methods could avoid imposing velocity term by the usage of moving coordinates. Inspired by them, we have proposed an FEM based method counting on Maxwell's equations in Lagrangian description to address the eddy current problems of deformable bodies and the coupling effect between EM fields and structural vibration [5]. Although effective, this new method lacks efficiency in term of simulation time.

In this paper, an adaptive mesh morphing method is proposed to deal with the time-consuming problem of our new method. Mesh morphing, coefficient matrix reformatting and time stepping procedure are presented.

II. NUMERICAL METHOD

A. Formulation of Lagrangian Approach

In the Largranigan description, the position and properties of particles are described in terms of referential coordinates, in which, spectators are attached to the deformable bodies and to describe all events from its material point of view [4]. The Lagrangian method is often employed for considering the geometric nonlinearity in structural analysis [6]. Similarly, in electromagnetic analysis, Lagrangian approach is also possible. For low frequency problems, Maxwell's equations in Lagrangian description can be written as

$$\boldsymbol{J} = Curl\boldsymbol{H}\,,\quad Div\boldsymbol{B} = 0$$
$$\frac{d\boldsymbol{B}}{dt} = Curl\boldsymbol{E} \qquad (1)$$

with the constitutive equations are

$$\boldsymbol{B} = \mu\boldsymbol{H}\,,\quad \boldsymbol{J} = \sigma\boldsymbol{E} \qquad (2)$$

where the uppercase letters ***B*, *H*, *E*, *J*** are described in the referential frame for magnetic flux density, magnetic field intensity, electric field intensity and current density; d/dt denotes the material derivative and Curl, Div are curl and divergence operators in the undeformed configuration respectively; σ is electric conductivity while μ is magnetic permeability. With Lagrangian description, the motional electromotive force is not explicitly expressed through the velocity term, but implicitly counted through Lagrangian coordinate system attached to material deformation.

Formally, Maxwell's equations in Lagrangian description are similar with governing equations of EM fields without deformable bodies. Therefore, the conventional A-ϕ formulation could be used directly. The Galerkin weak form of the governing equations are shown as follows

$$\int_{\Omega_c} \boldsymbol{N}\cdot\left(\frac{1}{\mu}\nabla\times\nabla\times\boldsymbol{A} + \sigma\frac{d\boldsymbol{A}}{dt} + \sigma\nabla\phi - \boldsymbol{J}_f\right)d\Omega = 0 \qquad (3)$$

$$\int_{\Omega_c} \boldsymbol{N}\cdot\nabla\cdot\sigma\left(\frac{d\boldsymbol{A}}{dt} + \nabla\phi\right)d\Omega = 0 \qquad (4)$$

where $\boldsymbol{N}$ is the shape function, $\boldsymbol{A}$, φ are magnetic vector potential and electric scalar potential respectively. Note that variables, including the shape function, are defined in Lagrangian coordinate system coincident with materials points. This means that when materials deform, the shape function needs to be altered according to material deformation. Correspondingly, finite element coefficient matrix should also be changed with material deformation. For code development, mesh and coefficient matrix for eddy current analysis should be updated to cope with material deformation.

B. Adaptive Mesh Morphing Method

Vibration induced eddy current can be automatically taken into account through updating EM mesh based on deformed structural configuration. Mesh morphing needs to be conducted at each time step in our previous paper. This leads to a big computational burden since mesh morphing and coefficient matrices reformatting are time-consuming. In this work, an adaptive mesh morphing method was proposed.

Structural deformation change is not always significant at each time step. In case of a step of small displacement, mesh morphing only slightly alters the coefficient matrices and thus contributes little to the motional electromotive force. Hence keeping mesh and coefficient metrics unchanged is reasonable in such a case. This allows us to do mesh morphing after several time steps once the total displacement becomes large. In this case, the step number to do mesh morphing can be selected according to the structural displacement and speed. On the other hand, the time step size is typically considered as a constant in the time integration scheme for transient analysis. However, for our Lagrangian approach, time stepping is associated with mesh morphing and matrices reformatting; thus, prolonging step size at appropriate situation means reducing the need for morphing and reformatting. Therefore, an adaptive time stepping procedure would be attractive. At last, it is possible to obtain the coefficient matrices of present time step from the former time steps through a linear interpolation approach if the deformation change is not large. In this way, the number of mesh morphing can be reduced.

Generally, because of magnetic damping effect, the amplitude of structural displacement and speed decay as time goes by. In the section of large structural speed, the step size is set to be small and morphing/reformatting is conducted at each step. Structural displacement/speed at subsequent steps is compared with the largest one; if it is an order of magnitude smaller than the largest one, the time step size is raised while the morphing step size is reduced. Finally, the local error is estimated and compared with a prescribed tolerance. If the local error is estimated to be too large, step size is remodified and computation at current step needs to be redone.

C. Coding Based on ANSYS Platform

Based on the Lagrangian approach and adaptive mesh morphing method, a code is developed on platform of ANSYS Parametric Design Language (APDL). The electromagnetic element SOLID236 is adopted to build the EM model while the structural element SOLID95 is used to build the mechanical model. The flowchart of the code is shown in Fig.1.

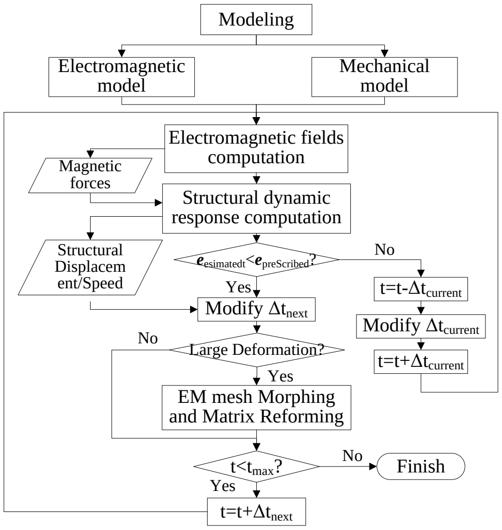

Fig.1 Flow chart of the developed code

III. NUMERICAL RESULTS

A benchmark problem for magnetic damping in torsional mode (Team problem 16), as shown in Fig.2, is given to demonstrate the feasibility of the proposed method. A copper plate is clamped at one end with a 27-turn exciting coil placed above it. Its length, clamped length, width and thickness are 115 mm, 10 mm, 40 mm and 0.3 mm respectively. A crossed time-varying field is produced by the coil with a current I=800(exp(-500t)-exp(-6000t))A, while an external steady magnetic field B_y along the y-axis is applied. The numerical results of dynamic response at point A (108 mm, 7.5 mm) for an external field B_y=0.3T are compared with experiment data[1] and the results of the conventional method [5], as shown in Fig. 3. Similar simulation results are obtained but with a reduced simulation time and fewer time steps.

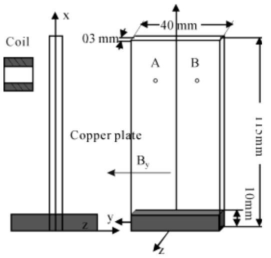

Fig.2 Schematic diagram of Team problem 16

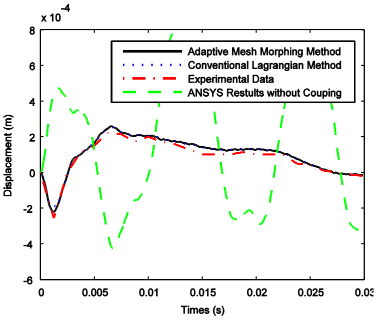

Fig.3 displacement response at point A, B_y=0.3 T

IV. CONCLUSIONS

In this paper, an adaptive mesh morphing method is proposed to deal with the time-consuming problem of the Lagrangian approach for treating coupling problem. Mesh morphing is only conducted once the contribution of motional electromotive force is large and step size is adjusted according to structural vibration. The numerical results of Team problem 16 demonstrated the validity of the proposed method.

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