# Numerical Investigations of the Effects of a High Magnetic Field on a Diamagnetic Yield Stress Fluid Flow — Opportunities of a Solid-gel Transition

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*Abstract*—The production of high magnetic fields offers new opportunities of magneto-mechanical interactions. In this paper, we investigate the possible effects of magnetic volume forces on a diamagnetic yield stress fluid flow. The numerical computation of magnetic volume forces requires a particular care for the magnetic field calculation. We study a magnetic configuration using two parallel surface current density. This configuration generates magnetic forces — usually used for magnetic levitation — which are able to modify the flow of a yield stress fluid. Two dimensionless numbers are introduced to define the necessary conditions to act on the fluid. Numerical investigations of the coupled problem are performed using a finite element software. We show the opportunity of a solid-gel transition in different parts of the fluid.

*Index Terms*—Fluid dynamics, magnetic levitation, stress, finite element methods.

## I. Introduction

Yield stress fluids have amazing mechanical properties. They can behave as solids under small applied stresses and flow like liquids above a certain load, called yield stress. These fluids have attracted considerable academic and industrial attention over the past few decades and their non-linear behaviour is widely used in many applications.It turns out that unambiguously determining the yield stress of a fluid from experiments is quite difficult. Part of the problem is due to the existence of a wall slip effect when acting on the fluid. We propose to use magnetic forces produced contactless by high magnetic fields to act on yield stress fluids. These forces have been successfully used to act on fluids [1]-[2]-[3] and we'd like to explore their effects on yield stress fluids flows.

## II. Exploring possibilities

### A. Magnetic, electrical and mechanical properties of typical water-based yield stress fluid

Bentonite is a smectite clay mineral with many applications that take advantage of its interesting rheological properties in water-based colloidal suspensions. When sufficiently concentrated, bentonite suspensions have a yield stress $\tau_y$ that results from the colloidal interactions among the particles. Bentonite is therefore an important component of materials such as drilling muds, paper coatings and pharmaceutical
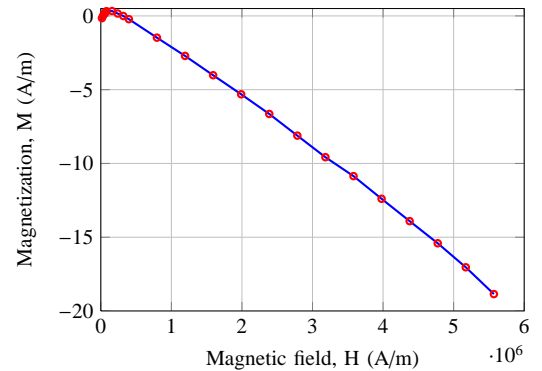


Figure 1: Magnetisation curve of water-based colloidal suspensions of bentonite.

products. The simplest model for this mechanical behaviour is the Bingham model (1) describing the evolution of the shear stress $\tau$ toward the shear rate $\dot{\gamma}$ with the help of the Bingham viscosity $\kappa$.

$$\begin{cases} \dot{\gamma} = 0 & \text{if} \quad \tau < \tau_y \\ \tau = \tau_y + \kappa\dot{\gamma} & \text{if} \quad \dot{\gamma} \neq 0 \end{cases} \qquad (1)$$

Bentonite suspensions show no particular electrical or magnetical properties. They behave mainly as a dielectric and diamagnetic fluid. The diamagnetism of the water outweighs the paramagnetic behaviour of the clay as shown in Fig. 1. The volume magnetic susceptibility is around $\chi \approx -3 \times 10^{-6}$.

### B. Magnetic forces

Magnetic forces acting on a fluid element of density $\rho$ can be derived from the Helmholtz free energy. According to [4], the magnetic forces experienced by a unit volume of an insulator fluid in a magnetic field $\boldsymbol{H}$ can be expressed as

$$\boldsymbol{F} = \frac{1}{2}\mu_0\chi\nabla\boldsymbol{H}^2. \qquad (2)$$

In magnetic levitation, magnetic forces are used to counteract the effects of gravity. In our case, there is no direct link between the magnetic forces and the shear stress generated

in the fluid. Numerical investigations of the coupled problem are necessary.

### C. Dimensional analysis

For an arbitrary unit volume of fluid in a gravitational field $\boldsymbol{g}$ and a magnetic field $\boldsymbol{H}$, the conservation of momentum leads to the general form of the Navier-Stokes equation for the velocity field $\boldsymbol{u}$

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right) = \rho \boldsymbol{g} + \nabla \cdot \boldsymbol{\sigma} + \frac{1}{2}\mu_0 \chi \nabla H^2, \qquad (3)$$

with $\boldsymbol{\sigma}$ the Cauchy stress tensor.

A dimensional analysis leads to

$$\mathrm{St}\frac{\partial \tilde{\boldsymbol{u}}}{\partial \tilde{t}} + \tilde{\boldsymbol{u}} \cdot \tilde{\nabla} \tilde{\boldsymbol{u}} = \mathrm{St}^2 \tilde{\boldsymbol{g}} + \tilde{\nabla} \cdot \tilde{\boldsymbol{\sigma}} + \frac{\mathrm{Ne}}{2}\tilde{\nabla}\tilde{H}^2. \qquad (4)$$

All tilded quantities are dimensionless and we introduce the usual Strouhal number St and a new dimensionless number Ne defined as

$$\mathrm{Ne} = \frac{\mu_0 \chi H_0^2}{\rho u_0^2}, \qquad (5)$$

where $H_0$ and $u_0$ are characteristic values of the magnetic field and the velocity field. Large values of |Ne| could led to a flow modification.

### III. NUMERICAL INVESTIGATIONS

### A. Computation of magnetic forces

Magnetic volume forces are computed with a finite element software using a potential vector formulation. Lagrange cubic finite elements are requested to obtain sufficient accuracy for the forces. In a 2D Cartesian configuration, with a potential vector $\boldsymbol{A} = A_z \boldsymbol{e}_z$, the forces are given by

$$\boldsymbol{F} = \frac{\chi}{\mu_0}\begin{pmatrix} \dfrac{\partial A_z}{\partial y}\dfrac{\partial^2 A_z}{\partial y \partial x} - \dfrac{\partial A_z}{\partial x}\dfrac{\partial^2 A_z}{\partial y^2} \\[2mm] -\dfrac{\partial A_z}{\partial y}\dfrac{\partial^2 A_z}{\partial x^2} + \dfrac{\partial A_z}{\partial x}\dfrac{\partial^2 A_z}{\partial x \partial y} \end{pmatrix}. \qquad (6)$$

### B. Magnetic configuration

Several configurations have been tested. We present an efficient configuration consisting of two parallel $xz$ infinite surfaces flown by two current densities $\boldsymbol{j_1}$ and $\boldsymbol{j_2}$ along $\boldsymbol{e}_z$. The below current density $\boldsymbol{j_2}$ is constant $J_0 = 10^5$ A/mm although the above current density $\boldsymbol{j_1}$ is modulated with a wavenumber $k = 50$ m$^{-1}$

$$\boldsymbol{j_1} = J_0 \sin(kx)\,\boldsymbol{e}_z, \qquad (7)$$

$$\boldsymbol{j_2} = J_0\,\boldsymbol{e}_z. \qquad (8)$$

### C. Fluid flow configuration

The non-linear behaviour of the yield stress fluid is well computed [5] by a regularised model [6]

$$\tau = \left(\kappa + \tau_y \frac{(1 - \exp(-m|\dot{\gamma}|))}{|\dot{\gamma}|}\right)\dot{\gamma}, \qquad (9)$$

introducing a parameter m = 100 s. The fluid flows between the two surface current densities.
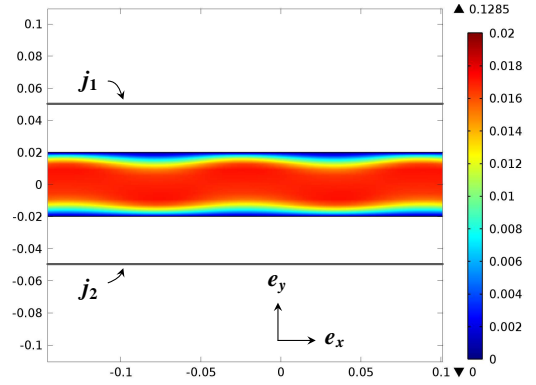


Figure 2: The velocity field (m/s) modifications of a yield stress fluid under high magnetic field.
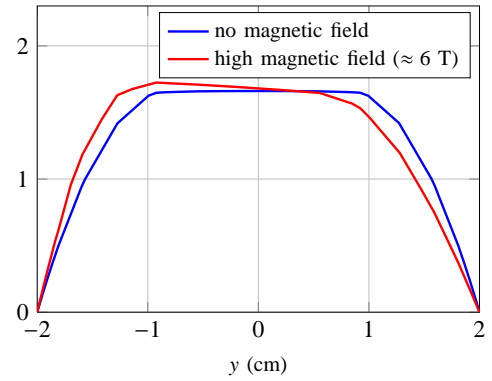


Figure 3: Velocity field (cm/s) with and without magnetic field.

### D. Results

For low values of the Bingham number Bn and large values of |Ne|, we observe changes in the velocity field of the fluid flow in Fig. 2 and Fig. 3. With L a characteristic length of the flow, the relevant dimensionless number to increase the effects of the magnetic forces is

$$\frac{|\mathrm{Ne}|}{\mathrm{Bn}^2} = \frac{|\chi|\mu_0}{\rho}\left(\frac{\kappa H_0}{L\,\tau_y}\right)^2. \qquad (10)$$

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