An Experimentally Based Mortar Cell Method Model for Electrical Interconnects

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***Abstract*—A 3D domain decomposition approach for analyzing electro-thermal contact problems is presented. The computational domain is subdivided into non-overlapping regions discretized according to the Cell Method, where variables and field equations are expressed directly in integral form suitable for coupling the contact problem to the electro-thermal one in the bulk regions. The electrical and thermal continuity between contacting regions is enforced by means of dual Lagrange multipliers as in mortar discretization methods. Contact resistance measurements are used to calibrate the parameters of the electrical constitutive equation, modeling voltage drops and power losses at the contact interface.**

***Index Terms*—Contact resistance, Electromagnetic modeling, Numerical simulation, Parameter estimation, Power dissipation.**

I. Introduction

The numerical simulation of the contact problems plays an important role in many industrial applications, but it is still challenging due to their complexity that is mainly due to multi-physics and multi-scale behavior and to geometry with non-convex and disconnected parts. Therefore, most of conventional FEMs are not suitable for their treatment [1]. Contacts can be much more effectively modeled by Domain Decomposition (DD), where the field problem is split into sub-problems matched by projection operators [2]-[4].
Among DD approaches, *mortar methods* provide a flexible way to enforce continuity between sub-domains by means of additional unknowns, i.e. the dual Lagrange multipliers [5]-[7].

In this paper a three-dimensional Mortar Cell Method for multi-scale modeling of electro-thermal contact problems is proposed, where the contact conductivity distribution is identified from experimental data so that contact resistance and local Joule losses are simulated.

II. Mortar Cell Method

A. Discrete contact interface problem

Contact is established when two members share a common interface where voltage drop occurs. According to [8] this discontinuity can be suitably modeled by a mortar formulation where interface conditions are imposed by means of dual Lagrange multipliers defined on the *mortar surface* Γ_m. This formulation is briefly outlined in the following.

In Fig. 1 the computational domain Ω is split into slave Ω_1 and master Ω_2 sub-domains, discretized with non-conforming meshes. Field problems are formulated with the Cell Method (CM) in terms of linear equations, where arrays of dofs such as line integrals and fluxes are defined on dual cell complexes.

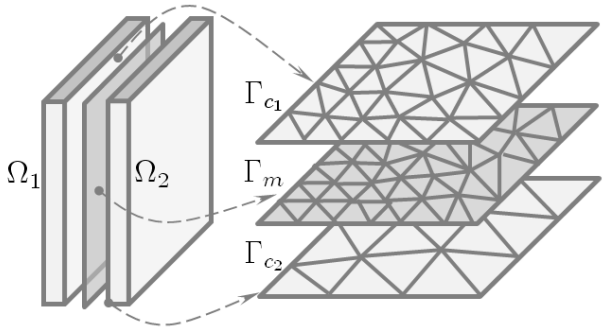

Fig. 1. Computational domain of the Mortar Cell Method.

Dual complexes on Γ_m are built from those on contacting surfaces Γ_{c1} and Γ_{c2}, yielding a finer mesh that allows discretizing the interface conditions accurately. This feature makes the mortar method more suited for multi-scale modeling than standard DDs, which do not make use of a mortar mesh.

Dual Lagrange multipliers consist of electric potentials $\boldsymbol{v}_m^{\pm}$ defined on the primal nodes of Γ_m and currents $\boldsymbol{j}_m$ defined on the dual faces of Γ_m. Potentials are mapped from Γ_m to Γ_{c1} and Γ_{c2} by means of a projection matrix $\boldsymbol{P}$. Currents are mapped from Γ_{c1} and Γ_{c2} to Γ_m by dual projection matrix $\tilde{\boldsymbol{P}} = \boldsymbol{P}^T$ according to the CM. Electric-thermal continuity between contacting parts is established by Kirchhoff's laws as in Circuit Theory. Kirchhoff's voltage law is expressed as

$$[\boldsymbol{v}_m] = \boldsymbol{P}\,\boldsymbol{v}_c \tag{1}$$

where $[\boldsymbol{v}_m] = \boldsymbol{v}_m^+ - \boldsymbol{v}_m^-$ is the array of the potential jumps and $\boldsymbol{v}_c$ is the array of the potentials on $\Gamma_c = \Gamma_{c1} \cup \Gamma_{c2}$. Kirchhoff's current law is imposed by means of the dual projection matrix

$$\tilde{\boldsymbol{P}}\,\boldsymbol{j}_m + \boldsymbol{j}_c = 0 \tag{2}$$

where $\boldsymbol{j}_m$ is the array of currents on Γ_m and $\boldsymbol{j}_c$ on Γ_c. The voltage drop across Γ_m depends on the contact resistance, which is accounted for by a surface conductivity σ_C (Sm^{-2}) whose parameters are inferred from experimental data (Section III). The constitutive equation becomes thus

$$\boldsymbol{j}_m = -\boldsymbol{M}_{\sigma_C}[\boldsymbol{v}_m] \tag{3}$$

where the matrix $\boldsymbol{M}_{\sigma_C}$, discretizing σ_C, is diagonal.

B. *Discrete formulation in bulk regions*

In steady-state conduction problems, CM degrees of freedom are voltages $\boldsymbol{u}$, defined on primal edges, and currents $\boldsymbol{j}$, defined on dual faces. In bulk regions the electric conductance operator $\boldsymbol{M}_\sigma$ is obtained by discretizing local Ohm's law $\boldsymbol{J} = \sigma(\theta)\boldsymbol{E}$ into $\boldsymbol{j} = \boldsymbol{M}_\sigma(\boldsymbol{\theta})\boldsymbol{u}$, where $\sigma(\theta) = \sigma_0/(1+\alpha\theta)$ and variables $\boldsymbol{\theta}$ are the temperature variations on primal nodes. Voltages are expressed directly in terms of potentials $\boldsymbol{v}$ by Kirchhoff's voltage law $\boldsymbol{u} = -\boldsymbol{G}\boldsymbol{v}$, where $\boldsymbol{G}$ is the discrete gradient operator. Kirchhoff's current law reads $\tilde{\boldsymbol{D}}\boldsymbol{j} + \boldsymbol{j}_c = \boldsymbol{j}_s$, where $\tilde{\boldsymbol{D}} = -\boldsymbol{G}^{\mathrm{T}}$ is the discrete divergence operator and $\boldsymbol{j}_s$ is the array of the source currents impressed on the boundary. By assembling constitutive and continuity equations the following non-linear matrix system is obtained:

$$\boldsymbol{G}^{\mathrm{T}}\boldsymbol{M}_\sigma(\boldsymbol{\theta})\boldsymbol{G}\,\boldsymbol{v} + \boldsymbol{j}_c = \boldsymbol{j}_s \tag{4}$$

where currents $\boldsymbol{j}_c$ link the bulk and mortar electric problems.

As regards the thermal problem, the stiffness matrix can be assembled as (4) with a constant matrix $\boldsymbol{M}_\lambda$ in place of $\boldsymbol{M}_\sigma$:

$$\boldsymbol{G}^{\mathrm{T}}\boldsymbol{M}_\lambda\boldsymbol{G}\,\boldsymbol{\theta} + \boldsymbol{q}_c = \boldsymbol{w}(\boldsymbol{v},\boldsymbol{\theta}) \tag{5}$$

where $\boldsymbol{q}_c$ are heat fluxes on the boundary and $\boldsymbol{w}$ is the array of electric powers dissipated inside dual cells. Electric and thermal equations are finally assembled into a single non-linear matrix system to be solved in terms of $\boldsymbol{v}$ and $\boldsymbol{\theta}$.

III. Contact Resistance Model

The conductivity distribution is modeled by the statistical relationship derived from Holm's theory, which describes local conduction phenomena [9]. Conductivity depends on apparent contact pressure *p*, surface roughness *r*, mean asperity slope *m*, bulk conductivity σ_b, and micro hardness *H*, as

$$\sigma_c = \frac{m\sigma_b}{2\sqrt{2\pi}r}\frac{\exp(-\xi^2/2)}{(1-\sqrt{p/H})^{1.5}} \tag{6}$$

where $\xi = \sqrt{2}\,\mathrm{erfc}^{-1}(2p/H)$. Geometric parameters *r* and *m* are evaluated from micro-mechanic measurements, while the dimensionless contact pressure *p/H* has to be identified from experimental data, according to [9].

Fig. 2 shows the test set-up: Plane-plane contact members (brass or aluminum, 9 cm diameter) have been considered for measuring contact resistance. When members are in contact, a DC current is impressed by a stabilized generator (6 kA max). The applied contact force and displacement are controlled by a universal machine (MTS 858 Mini Bionix II, 15 kN max). Electric potential, current, and temperature time profiles along electrodes have been registered by a HP 34970A multiplexer.

Fig. 3 shows preliminary measures for the brass specimens. A bath-tub force curve (14 kN max) is applied for 300 s and DC current is held constant in any test (200, 400, 600 A). It can be noted that contact resistance is almost independent on the impressed current and decreases with force till a minimum value of 7 μΩ is attained. An extended analysis of experiments and a comparison between measured and inferred contact resistance values are provided in the paper.

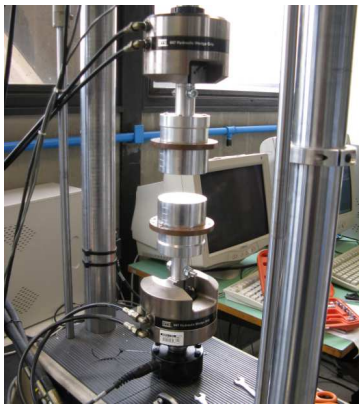

Fig. 2. Test set-up: plane-plane aluminum contact members.

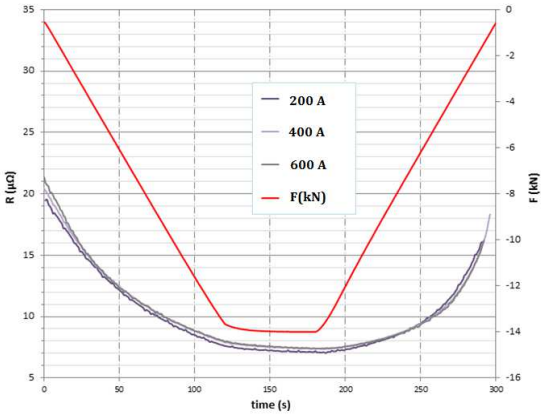

Fig. 3. Contact resistance and applied force vs. time (brass plane-plane members) for different impressed DC currents (200, 400, 600 A).

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