Turbo generators end windings magneto-mechanical analysis using a fully analytical magnetic model

A. A. Journeaux[1], F. Bouillault[1], O. Moreau[2]
[1]Laboratoire de Génie Électrique de Paris, 11 rue Joliot Curie, 91192 Gif-sur-Yvette, France
[2]EDF R&D, 1 Av. du Général de Gaulle, 92141 Clamart, France

***Abstract*—The present study describes a fully analytical expression of forces belonging to the end windings of large scale turbo generators. Such a magnetic model is of great help as air parts are no longer needed, thus heavily reducing the computation time and the memory cost. Forces are used to compute the associated mechanical model for which only the mesh of the wires is required. The location of the point of maximum stress is determined, and the influence on mechanics of zero divergence methods used in FEM codes will be studied.**

***Index Terms*—Turbogenerators, Vibrations, Magnetic forces, Magnetic fields, Analytic models.**

Introduction

Due to the growing demand for electric power, stresses applied to large scale generators is increasing. Moreover the existing generators have many years of use, thus raising the failure risk. Unfortunately, the limited surplus of power production tightens the availability of the machines for experiments leading to a high demand for accurate models of the end windings. Despite the progress in numerical computing, the ability to characterize such a system is limited. The complexity of the geometry, combined with the specific features required to properly model the generator, hardens the development of a representative model.
In an effort to set up a faithful description of the end windings, a massive three dimensional model of these conductors is presented. The aims are to analyze the influence of the current density distribution on mechanical values, as well as providing a prompt estimation of the force field load.

I. Magneto-mechanical model

Magnetic forces computation is a prerequisite for the mechanical problem resolution requirering the knowledge of the magnetic state of the system. A specific feature of this model is the resort to analytical formulae for the magnetic values, thus the single mechanical parts have to be meshed for a classical finite element model (FEM). Due to the intrinsic manufacturing process, the current density belonging to the conductors presents a uniform distribution. The copper parts are subdivided in independently isolated strands and, due to computational cost, cannot be modelled. Furthermore, because of the complex geometry of the conductor, usual methods used to impose the source current in FEM code strongly modify the distribution of the density. To overcome these difficulties, an analytical model has been developed.

A. Biot–Savart law

If the domain presents a uniform permeability, the magnetic induction $\mathbf{B}$ produced by a current density is expressed by:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu}{4\pi} \int_{\mathbf{r}' \in \Omega_c} \frac{\mathbf{J} \times (\mathbf{r} - \mathbf{r}')}{\|\mathbf{r} - \mathbf{r}'\|^3}, \tag{1}$$

where $\mathbf{r}'$ spans the whole domain of the inductor (Ω_c). $\mathbf{r}$ and $\mathbf{r}'$ are local positions in the spherical coordinates system. $\mathbf{J}$ is the current density belonging to the conductor, while μ is the (uniform) permeability of the domain. This expression can only be determined for simple geometries, and requires the decomposition of the windings into elementary blocks.

B. Formulaes for rectangular cross section inductors

L. Urankar has expressed [1], [2], [3] the Biot–Savart law for a 3D arbitrary length bar and a partial torus (cross section). Knowing the decomposition of every inductor into elementary bars and tori, the field can be computed. These expressions have resort to the Jacobi's elliptic integrals. Unfortunately, these expressions are singular for some positions making them unusable for the magneto-mechanical computation. As the geometry is complex, the main part of this study has been to determine the values of the integral (eq. 1) in every cases. The knowledge of the analytical magnetic field is of great help since the air box is no longer needed. Forces are computed using Lorentz expression:

$$\mathbf{f}(\mathbf{r}) = \mathbf{J}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}), \tag{2}$$

which have only to be determined within the conductors.

C. Magnetic scheme for forces computation

A geometrical description of the elementary bars and tori is automatically created, the main stages of the computation being:

- Extraction of the node belonging to the inductors;
- For each elementary block, geometrical transformation of positions of nodes in the local spherical coordinate system;
- Computation of the elementary magnetic induction and current density;
- Results gathering for all elementary pieces, thus the total magnetic induction is known;
- Force computation using the total current and total magnetic field.

Since these procedures are independent for all pieces, this computation is highly parallelizable. The geometry of the "twisted" part of the end windings has an analytical expression (involute of a cone), allowing the subdivision of this part into elementary bars for which the above method applies.

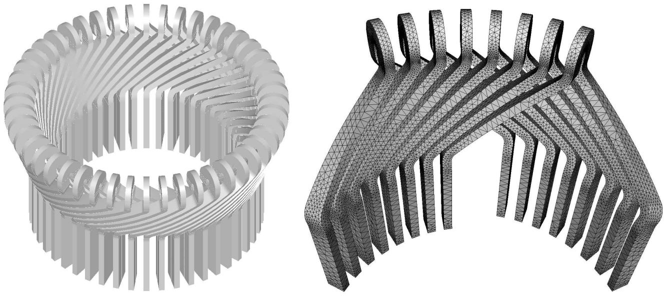

Figure 1: **Left:** Geometry used for the definition of the end winding inductors. **Right:** mesh used for the numerical application of the method.

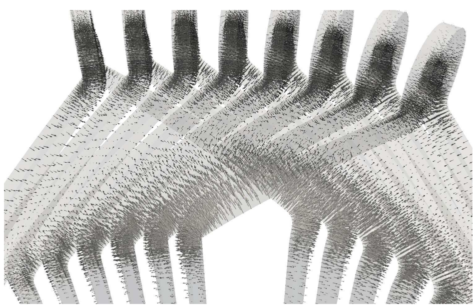

Figure 2: Forces computed using the present method

D. *Mechanical model*

We have used a linear elasticity model where the force density f_i arises from the analytical model (computed at nodes):

$$\sigma_{ij,j} + f_i = 0, \tag{3}$$

where σ is the Cauchy stress tensor. For elastic materials, stress, strain, and displacement are related by:

$$\sigma_{ij} = \lambda \, \mathrm{tr}(\epsilon_{ij})\delta_{ij} + 2\mu \, \epsilon_{ij} \;\text{ and }\; \epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{4}$$

where λ and μ are the Lam parameters, ϵ is the strain, and u is the displacement. The lower parts of the conductors are fixed to zero displacement, thus simulating a perfect embedding in the core of the generator.

II. Numerical results

The geometry of the end-windings is presented in figure 1. For the present test eight conductors have been modeled, and the corresponding result has been compared to a classical FEM computation (figures 2 and 4). Results appear in accordance with the one presented in refs. [4], [5].

Numerical application of the method: Figures 2 and 3 present the forces and the strain obtained using the analytical model. To ensure the normal component of the magnetic field to be null at the bottom face, a symmetric structure is virtually added.

Distribution of the current: Figure 4 illustrates the results obtained when inductors are not correctly modeled. Errors are mostly committed at the center of the top bend. At this position, force density is five times greater than the analytical one.

Figure 3: Total stress distribution issued from the mechanical resolution. High values are dark colored.

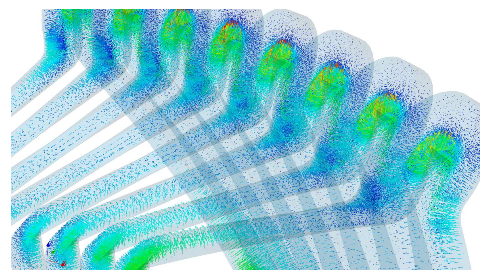

Figure 4: Forces obtained using FEM and massive conducting wires. Large errors are obtained at the top of the geometry leading to huge errors on the displacement.

Conclusion

With the aim to anticipate the impact of the rewinding of generators, and to provide help in understanding their aging, the demand for accurate models of generators end windings is increasing. This study presents a stage in the development of an accurate model. A fully analytical magnetic model has been developed to evaluate the impact of the distribution of the current density on the mechanical stresses. First results show that the use of a massive inductor is not acceptable since forces are erroneous. The analytical model presents the advantages to provide a first estimation of the magnetic force distribution, and significantly reduces the computational cost of the magnetic model.

References

[1] L. Urankar. Vector potential and magnetic field of current-carrying finite arc segment in analytical form, part i: Filament approximation. *Magnetics, IEEE Transactions on*, 16(5):1283 – 1288, sep 1980.

[2] L. Urankar. Vector potential and magnetic field of current-carying finite arc segment in analytical form, part ii: Thin sheet approximation. *Magnetics, IEEE Transactions on*, 18(3):911 – 917, may 1982.

[3] L. Urankar. Vector potential and magnetic field of current-carrying finite arc segment in analytical form, part iii: Exact computation for rectangular cross section. *Magnetics, IEEE Transactions on*, 18(6):1860 – 1867, nov 1982.

[4] Susumu Nagano, Tadashi Tokumasu, Masafumi Fujita, Yoshiyuki Inoue, Masayuki Ichimonji, Hitoshi Katayama, Daisuke Hiramatsu, and Tanzo Nitta. Study on vibration caused by electromagnetic force in the end portion of large-capacity cylindrical synchronous machines: Improvement of electromagnetic force estimation and verification of degradation on winding insulation. *Electrical Engineering in Japan*, 163(4):78–89, 2008.

[5] Ki-Chan Kim, Hyung-Woo Lee, Yon-Do Chun, and Ju Lee. Analysis of electromagnetic force distribution on end winding for motor reliance. *Magnetics, IEEE Transactions on*, 41(10):4072 – 4074, oct. 2005.