Coupled Magneto-mechanical Analysis in Isotropic Materials under Multi-axial Stress

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***Abstract*—In this paper, a model for the coupled analysis of magneto-mechanical problems in isotropic materials is proposed in which the effect of stress in transverse direction with respect to the flux density is also considered. To take account of the Villari effect in this case, magnetostriction data under transverse as well as longitudinal stress is required. The model uses the information included in the magnetostriction data and calculates the permeability variation including the anisotropy induced by the stress. The model is then applied to a simple two-dimensional problem, using simulated magnetostriction data, and the flux distribution in three cases of uncoupled, coupled without , and coupled with considering the transverse stress are compared.**

***Index Terms*—magnetostriction, coupled problems, stress, Villari effect.**

I. Introduction

It is a known fact that the magnetic behavior of ferromagnetic materials is affected by the mechanical stress occurring in the material [1]. The phenomenon which is the result of magneto-elastic energy balance in microscopic scale [2] is known as the Villari effect. Another phenomenon in ferromagnetic material, which is also due to the magneto-elastic energy balance [3], is the magnetostriction (MS) in which the material dimension changes in response to the magnetic flux density. The magnetostriction which is also affected by stress, contributes to the vibration and noise of electrical machines and apparatuses [4]. Therefore, the design of electrical machines, when the level of stress is relatively high, requires the numerical analysis of the problem taking account of the Villari effect and the effect of stress on MS. However, since both the Villari effect and MS depend on the same feature of the material [5], i.e. magneto-elastic energy balance, a model has been already proposed [6] using the knowledge of the magnetostriction behavior together with the zero-stress magnetization curve to describe the Villari effect. The model, however, neglects the effect of transverse stress on MS and consequently on the magnetization. Moreover, it was developed with the assumption that the stress does not alter the direction of the flux density. In general, the direction of the flux density is also affected by the stress, especially when applied in the transverse direction. In this paper, the model proposed in [6] is further developed to take account of the transverse stress and also the stress-induced anisotropy in the material. The proposed model requires the magnetostriction data of the material measured or modeled for transverse as well as longitudinal stress.

II. Theory

In a coupled magneto-mechanical system, the magnetic field strength, $\boldsymbol{H}(\boldsymbol{B},\boldsymbol{\sigma})$, which depends on the flux density $\boldsymbol{B}$, and the stress tensor $\boldsymbol{\sigma}$, is defined as follows [7]:

$$\boldsymbol{H}(\boldsymbol{B},\boldsymbol{\sigma}) = \frac{\partial}{\partial \boldsymbol{B}} w(\boldsymbol{B},\boldsymbol{\sigma}) , \tag{1}$$

where w is the total energy density of the system which is the sum of the magnetic and mechanical energy densities and can be written as follows [7]:

$$w(\boldsymbol{B},\boldsymbol{\sigma}) = \int_0^{\boldsymbol{B}} \boldsymbol{H}_0' \cdot d\boldsymbol{B}' + \frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}^{elast} , \tag{2}$$

where $\boldsymbol{H}_0 = \boldsymbol{H}(\boldsymbol{B},0)$ is the field strength corresponding to $\boldsymbol{B}$ under zero stress, and $\boldsymbol{\varepsilon}^{elast}$ is the elastic strain tensors in Voigt notation, related to the stress by Hook's law ($\boldsymbol{\sigma} = D\boldsymbol{\varepsilon}^{elast}$). Equation (1) then becomes

$$\boldsymbol{H}(\boldsymbol{B},\boldsymbol{\sigma}) = \boldsymbol{H}_0 + \frac{\partial}{\partial \boldsymbol{B}}\left(\frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}^{elast}\right) . \tag{3}$$

In [6], it is shown that

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{elast} + \boldsymbol{\varepsilon}^{ms}$$

where $\boldsymbol{\varepsilon}$ is the total strain tensor, and $\boldsymbol{\varepsilon}^{ms}$ is the *magnetostriction strain tensor*. The expression for $\boldsymbol{\varepsilon}^{ms}$ is given by the following equations [6]:

$$\boldsymbol{\varepsilon}^{ms} = \lambda \boldsymbol{S} , \tag{4}$$

$$S_{ij} = \frac{1}{2B^2}\left(3B_iB_j - \delta_{ij}B^2\right), \tag{5}$$

where λ is the magnetostriction parameter and $\boldsymbol{S}$ is the magnetostriction direction tensor. The partial derivative in (3) is taken under a constant ε . If magnetostriction is neglected, $\boldsymbol{\varepsilon}^{ms} = 0$, so the last term in (3) vanishes, making $\boldsymbol{H}$ independent of stress. Therefore, the stress dependency of $\boldsymbol{H}$, which shall be presented in this paper, arises from the magnetostriction phenomenon.

III. The Proposed Model

A. Magnetic Equation

By considering λ as a function of B, longitudinal stress $\sigma_{||}$ and the transverse stress $\sigma_{\perp}$, $\lambda = \lambda(B,\sigma_{||},\sigma_{\perp})$, equation (3) finally takes the following form:

$$\boldsymbol{H}(\boldsymbol{B},\boldsymbol{\sigma}) = \boldsymbol{H}_0 + \nu^{ms}(\boldsymbol{B},\boldsymbol{\sigma})\boldsymbol{B} , \tag{6}$$

where ν^{ms} is the MS reluctivity tensor representing the variation of reluctivity due to mechanical stress. Ampere's law in term of vector potential then becomes

$$\nabla \times \boldsymbol{H}(\boldsymbol{B},\boldsymbol{\sigma}) = \nabla \times \left((\nu + \nu^{ms})\nabla \times \boldsymbol{A}\right) = \boldsymbol{J}\,, \tag{7}$$

where $\nu = \nu(B,0)$ is the magnetic reluctivity evaluated from the zero-stress magnetization curve, $\boldsymbol{A}$ is the magnetic vector potential, and $\boldsymbol{J}$ is the current density. The derivation of ν^{ms} from equation (3) results in the following expressions:

$$\nu_{ij}^{ms} = \frac{3\lambda}{B^2}\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} + \begin{bmatrix} \nu_s^{ms} & 0 & 0 \\ 0 & \nu_s^{ms} & 0 \\ 0 & 0 & \nu_s^{ms} \end{bmatrix}, \tag{8}$$

$$\nu_s^{ms} = \frac{1}{B}\left(\frac{2\lambda}{B} - \left(1 - \frac{E}{1+\nu_P}\left(\frac{\partial\lambda}{\partial\sigma_{\parallel}} - \frac{1}{2}\frac{\partial\lambda}{\partial\sigma_{\perp}}\right)\right)\frac{\partial\lambda}{\partial B}\right)\boldsymbol{\sigma}\cdot\boldsymbol{S} + \frac{2\lambda}{B^2}tr(\sigma)\,, \tag{9}$$

where ν_s^{ms} is the *scalar* MS reluctivity, E and ν_P are Young's modulus and Poisson's ratio, and *tr*(.) stands for the trace of a tensor.

The finite element discretization of the coupled equations becomes

$$M\boldsymbol{A} = \boldsymbol{P}\,, \tag{10}$$

where M is the magnetic coefficient matrix, and $\boldsymbol{P}$ is the current source term.

B. Mechanical Equation

The mechanical equation of the proposed method is the same as in [6]. The discretized mechanical equation of the system is

$$K\boldsymbol{u} = \boldsymbol{f}^{\,m} + \boldsymbol{f}^{\,ms} \tag{11}$$

where K is the stiffness matrix, $\boldsymbol{u}$ is the displacement, and $\boldsymbol{f}^{\,m}$ and $\boldsymbol{f}^{\,ms}$ are the nodal magnetic and MS forces, respectively. The indirectly strong coupling method [6] was used for solving the coupled nonlinear equations (10) and (11).

IV. Results and Discussion

The model was applied to a simple 2D problem as shown in Fig1. An external stress of -5 MPa (compressive) is applied on the top of the model as shown in the figure. For the magnetostriction parameter a mathematical equation was assumed for the magnetostriction parameter as follows:

$$\lambda = 2\times10^{-6}(B^2 - 0.2B^4)\times(1 - 0.2\sigma_{\parallel} + 0.1\sigma_{\perp})\,.$$

Three analyses were carried out; an uncoupled analysis, a coupled analyses with transverse stress neglected, a coupled analyses with transverse stress considered. The results are shown in Fig. 2. Fig.2 (b) shows the flux density has been modified due to the longitudinal stress only. In Fig.2 (c) both longitudinal and transverse stress are considered. Fig.2 (d) shows the directions of $\boldsymbol{B}$ and $\boldsymbol{H}$ in the stressed regions. The anisotropy has been induced by transverse stress components.

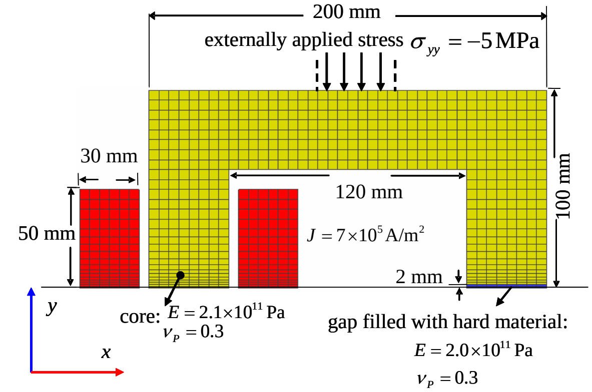

Fig. 1. Simple 2D reactor model analyzed.

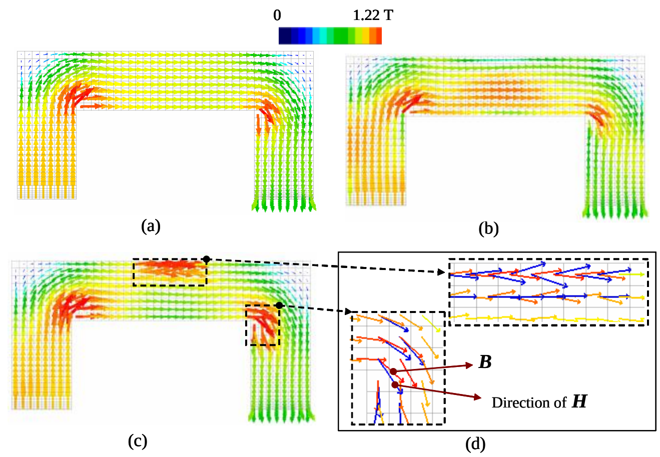

Fig. 2. (a) The flux distribution obtained from the uncouple analysis, (b) from the coupled analysis neglecting the transverse stress, (c) from the coupled analysis considering the transverse stress, (d) direction of ***H*** depicted by blue arrows beside ***B***-arrows (for better visibility, ***H***-arrows are scaled slightly longer than ***B***-arrows).

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