

# Coupled Analysis of Vibration Energy Harvesters Based on Nonconforming Voxel FEM

Takahiro Sato<sup>1</sup>, Kota Watanabe<sup>2</sup>, Hajime Igarashi<sup>1</sup>

<sup>1</sup>Graduate school of Information science and technology, Hokkaido University  
Kita 14, Nishi 9, Kita-ku, Sapporo, 060-0814, JAPAN

[tsato@em-si.eng.hokudai.ac.jp](mailto:tsato@em-si.eng.hokudai.ac.jp)

<sup>2</sup>Muroran Institute of Technology

**Abstract**—This paper presents a new coupled analysis method for vibration energy harvesters based on nonconforming voxel FEM (voxel-FEM). The coupled analysis of the harvester is performed using the staggered method, in which the electromagnetic field is analyzed using the voxel-FEM that can effectively model the moving mechanical system in the harvester without time-consuming mesh generations. It is shown from the coupled analysis that introduction of magnetic material in the coil improves the performance of the vibration energy harvester.

**Index Terms**—Electromagnetic induction, Energy harvesting.

## I. INTRODUCTION

Vibration energy harvester based on magnetic induction [1], [2] (called here harvester) is one of the energy harvesting devices, which transforms the vibration energy to electrical one through electromagnetic induction. The harvester can be regarded as the coupled system composed of electromagnetic field, mechanical, and electrical systems. To accurately estimate its performance, the coupled system has to be analyzed, for which the cantilever must be modeled considering its motion into account. The difficulty in the coupled analysis of the harvester exists in the dynamic generation of the 3D FE model of the moving objects.

In this paper, the coupled analysis method based on the nonconforming voxel FEM (voxel-FEM) [3] for the harvester is introduced, in which Maxwell, motion, and circuit equations are solved alternatively using the staggered method [4]. The present voxel-FEM can effectively model the moving mechanical system in the harvester without time-consuming mesh generations. Thanks to this feature, the present method can resolve the difficulty in the coupled analysis of the harvester, and the output power of the harvester can effectively be estimated. It is shown that the computed magnetic force and electromotive force (EMF) in the voxel-FEM are in good agreement with those obtained by the conventional tetrahedral FEM which involves mesh generation. It is found from the numerical results that introduction of magnetic material in the coil improves the performance of the harvester.

## II. COUPLED ANALYSIS OF VIBRATION ENERGY HARVESTERS

The harvester based on the magnetic induction consists of a pickup coil and magnets fixed to the thin cantilever as shown in Fig.1. The harvesters would be placed on the vibrating objects such as engines, motors and bridges. The ambient vibration makes the magnet have sinusoidal

displacements relative to the coil. As a result, the magnetic flux across the coil changes in time and consequently EMF is induced. Furthermore, the magnetic force between the coil and magnets changes the vibration of the magnets.

In this work, we analyze the harvester shown in Fig. 1 which contains the magnetic material in the coil. This model has two pole-pairs which has been proposed in [2], [3]. The magnetic flux across the coil changes depending on the relative position because each pair has different magnetization direction. Moreover, magnetic material is introduced in the coil to increase the magnetic flux across the coil. The cantilever would have complicated motion due to the magnetic force between the coil and magnets.

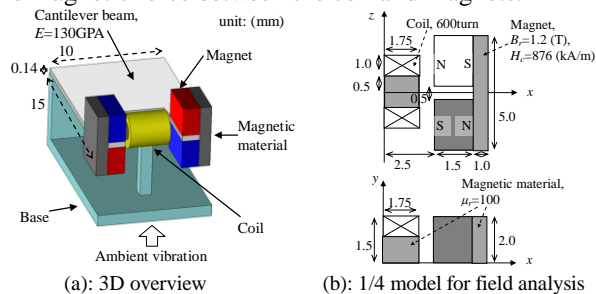


Fig. 1. Vibration energy harvester model

### A. Coupled analysis

The cantilever is modeled as a simple spring damper system for simplicity, and the external circuit connected to the coil is one load,  $r$ . The governing equations of the harvester are given by

$$m\ddot{p} + c_m(\dot{p} - \dot{q}) + k(p - q) = F - mg, \quad (1)$$

$$\nabla \times v(\nabla \times \mathbf{A}) = \mathbf{J} + v_0 \nabla \times \mathbf{M}, \quad (2)$$

$$\frac{D\Phi}{Dt} + rI = 0, \quad (3)$$

where  $m$  is mass of objects on the tip of the beam,  $c_m$  and  $k$  are damping and effective stiffness coefficients of the beam.  $p$  and  $q$  (m) are magnets and coil positions, respectively, and  $F$  (N),  $\mathbf{J}$  are the magnetic force acting on the magnets and the current density, respectively. The current  $I$  in (3) is obtained by integrating  $\mathbf{J}$  on the cross-section of the electric circuit. In addition,  $D/Dt$  is Lagrange derivative to compute EMF due to the coil movement [5]. In this work, (1) and (2) are solved using the Runge-Kutta method and the voxel-FEM, respectively.

To make effective modeling of the relative displacement between the coil and magnets, the FE analysis is performed

using two nonconforming voxel meshes, as shown in Fig. 2. The mesh1 and mesh2 correspond to the FE models of the coil and magnets respectively. The nodal positions in mesh1 are changed corresponding to their relative displacement  $q-p$ . On the interface between these meshes, the nonconforming technique is employed, that is, the unknown assigned to an edge,  $e_s$ , on the interface in the mesh1 is interpolated by those in the mesh2,  $A_i$ , as follows [4]:

$$A_s = \sum_{i=1}^{N_{ms}} A_i \int_{e_s} N_i \cdot dl \equiv \sum_{i=1}^{N_{ms}} c_{i,s} A_i, \quad (4)$$

where  $N_i$ ,  $l$ ,  $N_{ms}$  are the interpolation function of edge  $i$ , tangential vector of  $e_s$ , the number of the master edges on the interface in mesh1, respectively. The Lagrange coordinate system is here employed to describe the motion of mesh1. Hence, Lagrange derivative in (3) can simply be computed from

$$\frac{D\Phi}{Dt} = \frac{1}{\Delta t} \mathbf{b}_u \cdot (\mathbf{C}A_n - \mathbf{C}A_{n-1}), \quad (5)$$

where  $\Delta t$  is time increment,  $\mathbf{b}_u$  is the right hand vector when the forced current is 1A,  $A_n$  and  $A_{n-1}$  are solution vectors on master edges obtained at time steps  $n$  and  $n-1$ , respectively. In addition,  $C$  is an  $N_M \times (N_M + N_S)$  matrix whose components are

$$C(i, j) = \begin{cases} 1 & j \text{ is master edge and } i = j \\ c_{i,j} & \text{otherwise} \end{cases} \quad (6)$$

where  $N_M$  and  $N_S$  are the number of master and slave edges, respectively.

The coupled analysis is performed using the staggered method [4]. Namely, (1), (2), and (3) are solved alternatively until convergence at each time step  $n$ . This inner iteration is here called sub-cycle. The discretized governing equations are given by

$$\dot{u}_n^k = -\frac{c_m}{m} (u_n^{k-1} - \dot{q}_n) - \frac{k}{m} (p_n^{k-1} - q_n) + \frac{F_n^{k-1}}{m} - g, \quad (7)$$

$$\dot{p}_n^{k-1} = u_n^k \quad (8)$$

$$\mathbf{C}^T \mathbf{K} \mathbf{C} A_n^k = \mathbf{C}^T \mathbf{b}_n^{k-1}, \quad (9)$$

$$I_n^k = -\frac{1}{r \Delta t} \mathbf{b}_u^T \mathbf{C} (A_n^{k-1} - A_{n-1}), \quad (10)$$

where  $k$  is the step number in the sub-cycle, and  $\mathbf{b}_n^k$  is the FE vector corresponding to  $I_n^k$ .

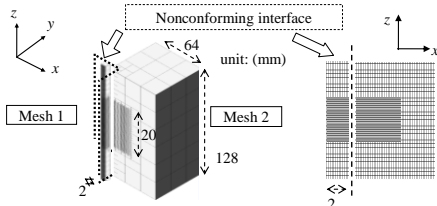


Fig. 2. Two nonconforming voxel meshes.

### B. Numerical results

First, the computational accuracy in the voxel-FEM is evaluated by analyzing  $F$  and EMF assuming that the time change in  $q-p$  is constant, +0.5 mm/s. Note that the coupling between the electromagnetic and mechanical systems is not

taken into account. Fig. 3 shows the changes in  $F$  and EMF, from which we can see that  $F$  and EMF computed in the voxel-FEM are in good agreement with those in the conventional tetrahedral FEM. Thus, the accuracy in the voxel-FEM is satisfactory for the coupled analysis.

Next, the coupled analysis of the harvester is performed. In this analysis,  $m$ ,  $c_m$ ,  $k$ ,  $r$  are set to 1 g, 0.025 Ns/m, 280 N/m, 100  $\Omega$  respectively, and the vibration whose amplitude and frequency are 0.1 mm and 50Hz, respectively, is applied to the coil. The analysis results are shown in Fig. 4. The changes in the magnets positions are shown in Fig. 4(a), from which we can see that the amplitude of magnets' oscillation is about 1mm while the input amplitude is 0.1mm. The reason for the large displacement is attributed to that fact that strong magnetic force, as shown in Fig. 4(c), changes the behavior of the magnets oscillation. The analysis results also show that EMF in the harvester in which magnetic material is introduced in the coil is much greater than that without magnetic material as shown in Fig. 4(b). This result suggests that magnetic material in the coil increases the output in the harvester. This improvement would be attributed to introduction of the magnetic material which makes closed flux pass.

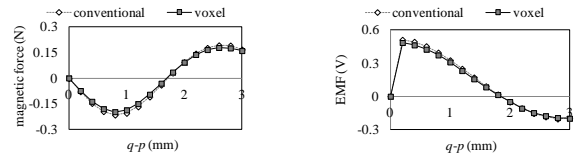


Fig. 3. Computed magnetic force and EMF.

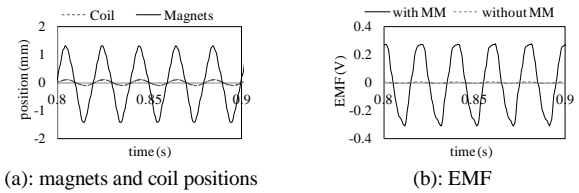
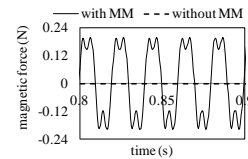


Fig. 4. Coupled analysis results of harvester.



(c): magnetic force

Fig. 4. Coupled analysis results of harvester.

### REFERENCES

- [1] S. P. Beeby, R. N. Torah, *et al.*, "A micro electromagnetic generator for vibration energy harvesting", *J. Micromech. Microeng.* Vol. 17, pp. 1257-1265, 2007.
- [2] R. Torah, *et al.*, "Self-powered autonomous wireless sensor node using vibration energy harvesting", *Meas. Sci. Technol.* Vol. 19, pp. 1-8, 2008.
- [3] T. Sato, K. Watanabe, H. Igarashi, "Accuracy Evaluation of Three Dimensional Finite Element Analysis Based on Nonconforming Voxel Meshes", *COMPEL*, Vol. 34, No. 2, 2015.(to be published)
- [4] T. Niho, T. Horie, Y. Tanaka, "Numerical Instability of Magnetic Damping Problem of Elastic Plate", *IEEE Trans. Magn.*, Vol. 36, No. 4, pp. 1373-1376, 2000.
- [5] K. Muramatsu, *et al.*, "Three-Dimensional Steady-State Eddy-Current Analysis of Moving Conductor Using Edge Elements and Moving-Coordinate System", *IEEE Trans. Magn.*, Vol. 38, No. 2, pp. 597-600, 2002.