Low-Frequency Time-Domain On-Surface Radiation Boundary Condition for Scattering Applications

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***Abstract*—An on-surface radiation boundary condition procedure (OSRBC) in the time domain is presented. The method extends well-known OSRBCs originally developed for high frequencies (short time transients) to low frequencies or long-time transients in the time-domain. Starting with asymptotic approximations we obtain an operator that is uniformly valid in time. Thus, the OSRBC developed here is valid at low and high frequencies. Its numerical implementation and its performance in low frequency problems are demonstrated through examples of scattering from conducting cylinders. The advantage of the new OSRBC is that it is applicable to a wide range of frequencies it can be applied to short and long time transients.**

***Index Terms*—Scattering, radiation boundary conditions, time-domain analysis, numerical analysis.**

I. Introduction

The combination of asymptotic and numerical methods has led to efficient modeling of scattering of electromagnetic waves in open domains. The success of these methods is mostly due to the radiation series operators of Bayliss and Turkel [1]. The series operators B_n ensure the cancellation of the first n terms of the asymptotic scattered field series as $r \to \infty$. These far-field operators were used as on surface radiation boundary conditions (OSRBCs) and combined with various numerical methods to obtain approximate solutions for scattering from convex targets [2,3]. The advantage of this approach is that now the field and its normal derivatives can be obtained from the approximate condition and the integral equations are reduced to integration over the surface of the scatterer (or on its contour for a 2D object).

This approach has been limited to high frequency problems, primarily in the frequency domain because OSRBCs are based primarily on the 1st order Baylis and Turkel operator which is a high frequency operator. There is however a need to develop operators in the time domain to tackle problems in low frequency scattering for a variety of applications including electromagnetic pulse (EMP) simulations and target identification. To this end, a variable time-dependent coefficient operator capable of simulating the response over a wide frequency range has been investigated in [4] and this operator forms the basis of the OSRBC proposed in this work. In essence, we start with the Baylis and Turkel operator and modify it to include low frequency components. By so doing the new OSRBCs are applicable over a wide frequency range, capable of simulating both low and high frequency fields. As such the new OSRBC is well suited for solution of long transient problems.

II. The On Surface Radiation Boundary Condition

We assume here a TM wave incident on a cylinder with its cross-section in the x-y plane and its axis along z. The incident and scattered fields are:

$$\vec{\mathrm{E}}_{inc} = U_{inc}\left(\vec{x},t\right), \quad \vec{\mathrm{E}}_{s} = U_{s}\left(\vec{x},t\right)\hat{\mathbf{z}} \tag{1}$$

Both fields satisfy the homogeneous wave equation. We represent the solution using the Stratton-Kirchoff formula [5] in the following form:

$$\bar{U}_s\left(\vec{x},k\right) = \int_\Gamma \left[G(\vec{x},\vec{x}')\frac{\partial \bar{U}_s\left(\vec{x}'\right)}{\partial n'} - \bar{U}_s\left(\vec{x}'\right)\frac{\partial G(\vec{x},\vec{x}')}{\partial n'} \right] ds' \tag{2}$$

where $G(\vec{x},\vec{x}')$ is the free space Green's function, $\partial/\partial n'$ is the outward normal derivative on Γ, $\vec{x}$ is the field point and $\vec{x}'$ the source point on the boundary contour. It is then obvious that the main issue is the calculation of the normal derivative $\partial\bar{U}_s\left(\vec{x}\right)/\partial n'$ on Γ since for conducting contours $\bar{U}_s$ can be calculated from the incident field. To do so we start with B_1 and an integral form of B_2, written in the time domain [4]:

B_1:
$$\frac{\partial U_s}{\partial r} = -\frac{\partial U_s}{\partial t} - \frac{1}{2r}U_s \tag{3}$$

B_2:
$$\frac{\partial U_s}{\partial r} = -\frac{\partial U_s}{\partial t} - \frac{1}{2r}U_s + \frac{1}{2}\int_{t_i}^{t}\left(\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} + \frac{1}{4r^2}\right)U_s e^{-(t-\tau)/r}d\tau + \Delta E_i \tag{4}$$

where $\Delta E_i = e^{-(t-t_i)/r}\left(\partial U_s/\partial r + \partial U_s/\partial t - U_s/2r\right)_{t=t_i}$ is found from the initial condition.

A new OSRBC based on the operator in [4] which we call the HHM operator is proposed here. Since the HHM operator corrects for the low frequency behavior and is based on asymptotic analysis as well as it has the correct behavior at large distances for both low and high frequencies. It has the following form for cylindrically symmetric waves:

$$\left(\frac{\partial U_s}{\partial t} + \delta_0(t)\right)\left(\frac{\partial U_s}{\partial r} + \frac{\partial U_s}{\partial t} + \frac{U_s}{2r}\right) - \frac{U_s}{8r^2} = 0 \tag{5}$$

where $\delta_0(t) = 1/(4r(1-2rG_0(t))$, $G_0(t) = 1/(r\ln((2t+D)/r))$ and $D > re^2/2$. D is an integro-differential operator. The expression in (5) can be seen as the B_1 operator in (3) operated upon by $\partial/\partial t + \delta_0(t)$ to which an additional term $U_s/8r^2$ was added. Analogous expressions of the operator for nonsymmetric waves also exist [4]. Based on this operator we construct the new HHM OSRBC as follows (using here the general form $\partial/\partial n'$ instead of $\partial/\partial r$ for the cylindrical case):

$$\frac{\partial U_s}{\partial n'} = -\frac{\partial U_s}{\partial t} - \frac{k\left(x',y'\right)}{2}U_s + \int_{t_i}^{t}\left(\frac{1}{2}\frac{\partial^2 U_s}{\partial s'^2} + \frac{\kappa^2\left(x',y'\right)}{4}U_s\right)e^{-\int_\tau^t \delta_0(t')dt'}d\tau + \Delta E_i \quad (6)$$

where $\kappa\left(x',y'\right)$ is the curvature, s' the tangential variable and $\Delta E_i = e^{-\int_{t_i}^{t}\delta_0(t')dt'}\left(\partial U_s/\partial n' + \partial U_s/\partial t - \kappa\left(x',y'\right)U_s/2\right)$ is determined again from the initial conditions.

III. Numerical Implementation

For the perfect conductor case discussed here, $U_s = U_{inc}$ on Γ and the normal derivative can be obtained from (6) as:

$$\frac{\partial U_s}{\partial n'} = -\frac{\partial U_{inc}}{\partial t} + \frac{\kappa\left(x',y'\right)}{2}U_{inc} + I(t) + \Delta E_i \quad (7)$$

where:

$$I(t_1) = \int_{t_i}^{t_1}\left(\frac{1}{2}\frac{\partial^2 U_{inc}}{\partial s'^2} + \frac{\kappa^2\left(x',y'\right)}{8}U_{inc}\right)e^{-\int_\tau^{t_1}\delta_0(t')dt'}d\tau . \quad (8)$$

In this case, the normal derivative can be evaluated through time marching methods. For $t_m = t_i + m\Delta t$ and Δt small:

$$I(t_{m+1}) = e^{-\Delta t\delta_0(t_{m+1}-\Delta t/2)}I(t_m) + \Delta t e^{-(\Delta t/2)\delta_0(t_{m+1}-\Delta t/4)}\left(\frac{1}{2}\frac{\partial^2 U_{inc}}{\partial s'^2} + \frac{\kappa^2\left(x',y'\right)}{8}U_{inc}\right)_{t=t_{m+1}-\Delta t/2} . \quad (9)$$

This then allows calculation of the normal derivative at the surface of the conductor and from it other quantities such as the scattering cross-section. For verification purposes, an exact solution is found for the conducting cylinder using potential theory [5]. Equations (5), (6) are also evaluated numerically using the method described above for the purpose of comparison with the new OSRBC.

It should be noted that the OSRBC developed here is not limited to perfect conductors. The method described can be extended to imperfect conductors and dielectrics.

IV. Some Results

To demonstrate the usefulness and effectiveness of the OSRBC developed here we use a conducting circular cylinder of radius $r_0 = 1$. The incident electromagnetic wave is a TM Gaussian impulse plane wave. We obtain results at low and high frequencies and compare them with the exact solution as well as with the B_1 and B_2 Bayliss and Turkel series. Here we only show the normal derivatives of the field but the scattering cross section has also been computed as were the fields and scattering cross-section of other geometries. Figure 1 compares the normal derivatives at $\theta = 90^\circ$ (grazing angle) for the high frequency case $k_0 = 5$. The HHM OSRBC is clearly close to the exact solution. B_2 also performs relatively well but the results using B_1 are rather poor. These results are somewhat sensitive to the location at which the derivatives are calculated since all boundary conditions are based on the geometric theory of diffraction. Figure 2 compares the results for the same conditions but for the low frequency case $k_0 = 0.001$. In this case the normal derivative is independent of the incidence direction due to the low frequency diffraction phenomenon. The results in Figures 1 and 2 demonstrate that the HHM OSRBC is a uniform time-domain OSRBC that is valid for both high and low frequencies.

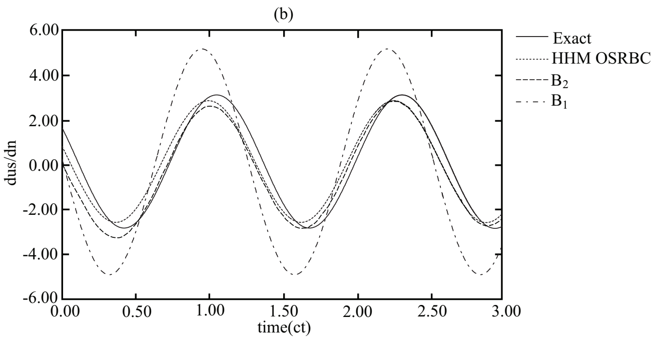

Fig. 1. Comparison of OSRBCs and potential theory solutions for normal derivatives on a conducting circular cylinder at $\theta = 90^\circ$ (grazing angle), at high frequency, k=5.

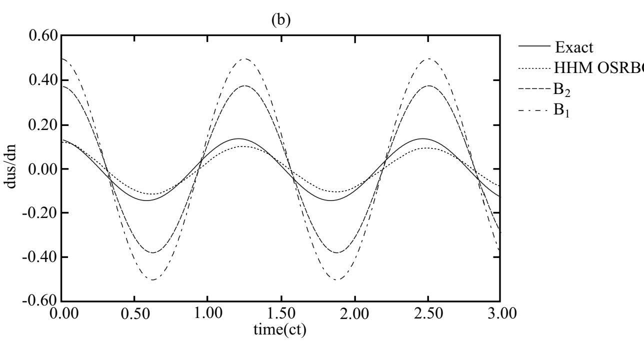

Fig. 2. Comparison of OSRBCs and potential theory solutions for normal derivatives on a conducting circular cylinder at $\theta = 90^\circ$ (grazing angle) at low frequency, k=0.001.

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