

An Integral Formulation for the Computation of 3D Eddy Current Using Facet Elements

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Abstract—In this paper, we present a volume integral formulation to compute eddy currents in non-magnetic conductors using facet elements. In the approach, the conductors are meshed by finite elements. Each mesh element is then considered as a lumped element of an electrical equivalent circuit. An algorithm detecting the independent loops is finally used for the circuit resolution. The formulation is validated thanks to a multiply connected regions problem.

Index Terms— Approximation methods, eddy currents, Integral equation method.

I. INTRODUCTION

Facet elements have been developed to model magnetostatic problems for the finite element method [1] and for the integral equations method [2], [3]. Using the facet interpolation, different unknown can be approximated, such as the current in [1], [3] and the magnetization in [2]. This approach seems very powerful as it allows both incorporating different geometrical structures and lumped elements of electrical network within a unified integral formulation.

The PEEC method (Partial Element Equivalent Circuit) is mainly used for the modeling of complex interconnection and can be applied to a large range of devices where the air region is dominant [4]. However, the classical PEEC method does not enable the 3D modeling of conductive.

This paper proposes a generalisation of the classical inductive PEEC formulation using facet elements. An equivalent electrical circuit, whose branches are the facets and the nodes of these branches are the centroid of elements of mesh, is proposed.

II. THE INTEGRAL FORMULATION

A. Problem description

Eddy current in conductive materials fit the classical PEEC formulation which is derived from the equation governing the total electric field at a point r inside the conductors:

$$\frac{\mathbf{J}}{\sigma} + j\omega \frac{\mu_0}{4\pi} \int_{\Omega_c} \frac{\mathbf{J}}{r} d\Omega_c = -\mathbf{grad}V \quad (1)$$

where Ω_c is conductive region, \mathbf{J} is the current density, V is the scalar electric potential, σ is the material conductivity, μ_0 is the vacuum permeability, and ω is the excitation pulse.

B. System assembly

With facet elements interpolation, the current density is described as follows:

$$\mathbf{J} = \sum \mathbf{w}_f \cdot \mathbf{I}_f$$

where \mathbf{w}_f is shape function and \mathbf{I}_f is flux across the f th facet. The normal component of \mathbf{w}_f is conserved.

Applying the Galerkin method to (1), a system of linear equations is obtained:

$$[\mathbf{Z}_b] \cdot \{\mathbf{I}_f\} = \{\mathbf{U}_b\} \quad (2)$$

with $[\mathbf{Z}_b] = [\mathbf{R}] + j\omega \cdot [\mathbf{L}]$

$$R_{fg} = \int_{\Omega_c} \frac{\mathbf{w}_g \cdot \mathbf{w}_f}{\sigma} d\Omega_c$$

where:

$$L_{fg} = \frac{\mu_0}{4\pi} \int_{\Omega_c} \mathbf{w}_g \cdot \int_{\Omega'_c} \frac{\mathbf{w}_f}{r} d\Omega'_c d\Omega_c$$

$$\text{and } U_{bg} = - \int_{\Omega_c} \mathbf{w}_g \cdot \mathbf{grad}V d\Omega$$

Matrix $[\mathbf{Z}_b]$ can be seen as the impedance matrix of the electrical equivalent circuit generated, $[\mathbf{R}]$ is matrix of the resistance terms, and $[\mathbf{L}]$ is matrix of the mutual inductance terms. To avoid the singularity problem of $[\mathbf{L}]$, we assume that the face to face interaction of the element onto itself equal to zero.

We consider now the following equality:

$$\begin{aligned} \int_{\Omega_c} \mathbf{w}_g \cdot \mathbf{grad}V d\Omega &= \int_{\Omega_g} \text{div}(\mathbf{w}_g \cdot V) d\Omega_c - \int_{\Omega_g} \text{div}(\mathbf{w}_g) \cdot V d\Omega \\ &= \underbrace{\int_{\Gamma} (\mathbf{w}_g \cdot V) \cdot \mathbf{n} d\Gamma}_{(I)} - \underbrace{\int_{\Omega_i} \text{div}(\mathbf{w}_g) \cdot V d\Omega}_{(II)} \end{aligned}$$

where \mathbf{n} is the outward normal vector on the boundary. The first term (I) vanishes inside the domain because $\mathbf{w}_g \cdot \mathbf{n}$ is conserved and on the boundary, since the current is tangential. Moreover, (I) allows imposing V on a surface boundary.

The second term (II): $\text{div}(\mathbf{w}_g) = \pm \frac{1}{v}$ (v is volume of element). We get then:

$$\int_{\Omega_c} \mathbf{w}_g \cdot \mathbf{grad}V d\Omega = \pm (V_a - V_b)$$

$$\text{with } V_a = \int_{\Omega_a} V d\Omega_a$$

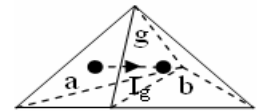


Fig.1. Orientation of facet

The sign (\pm) depends on the orientation of facet. For example, in Fig.1, the facet g is oriented from the element a to the element b .

C. Resolution of the electrical circuit

It is not possible to solve directly the system of linear equations (2), because equations are not independent The

current density has to satisfy the several conditions. In the conductive region Ω_C :

$$\text{div } \mathbf{J} = 0 \quad (3)$$

and on its boundary $\partial\Omega_C$

$$\mathbf{n} \cdot \mathbf{J} = 0 \quad (4)$$

The current flux is represented by the facets. The facets are considered as the branches of an electrical circuit. The conditions of (3) and (4) will be ensured when applying a loop analysis.

By using the independent loops search proposed in [5], we can write a new system of linear equations where the unknowns are the currents flowing in the loops :

$$[\mathbf{M}] \cdot [\mathbf{Z}_b] \cdot [\mathbf{M}^t] \cdot \{\mathbf{I}_m\} = [\mathbf{Z}_m] \cdot \{\mathbf{I}_m\} = [\mathbf{M}] \cdot \{\mathbf{U}_b\} = \{\mathbf{U}_s\} \quad (5)$$

where $[\mathbf{Z}_b]$ is a complex branch impedance matrix, $[\mathbf{Z}_m]$ is a complex loop-based impedance matrix, $\{\mathbf{I}_m\}$ is a vector of independent loop-based currents, $[\mathbf{M}]$ is the incidence matrix (branch – fundamental independent loop matrix) where the value of each element can be -1, 0 or 1, $\{\mathbf{U}_s\}$ is the vector of source voltages (most of the time is equal to 0), and the size of $\{\mathbf{I}_m\}$ is the number of fundamental loops. Once the problem is solved, we can obtain the currents flowing on each branch by the following relationship:

$$\{\mathbf{I}_b\} = [\mathbf{M}^t] \cdot \{\mathbf{I}_m\}$$

III. NUMERICAL EXAMPLE

In order to validate the proposed formulation and to show its performances, we consider an example with a multiply connected region. The results will be compared to those obtained with FLUX [6], a commercial Finite Element Method (FEM) program.

In this example, we compute the eddy currents in a conductive disk with a hole ($\sigma = 6e+7$ S/m) which is placed generated by a loop conductor (Fig.2). This conductor is fed by a current source.

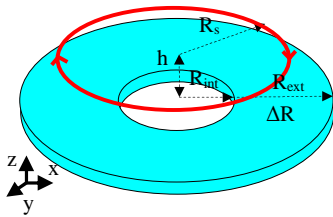


Fig.2. Geometry considered

In this case, with the presence of current source, the equations in system (2) need to be adapted. We must add a mutual term:

$$[\mathbf{Z}_b] \cdot \{\mathbf{I}_f\} + [\mathbf{Mu}] \cdot \{\mathbf{I}_s\} = \{\mathbf{U}_b\} \quad (6)$$

where $\{\mathbf{I}_s\}$ is current flowing in the conductor and $[\mathbf{Mu}]$ is the matrix of mutual inductance between the circuit loop and each facet element:

$$\text{Mu}_{ik} = j\omega \frac{\mu_0}{4\pi} \int_{\Omega_C} \mathbf{w}_i \cdot \int_{\Omega_S} \frac{\mathbf{t}_{0k}}{r} d\Omega_S d\Omega_C$$

where \mathbf{t}_{0k} is unit vector defining the current direction. Note that for the case $\{\mathbf{U}_b\} = \{\mathbf{0}\}$, (6) becomes:

$$[\mathbf{Z}_b] \cdot \{\mathbf{I}_f\} = -[\mathbf{Mu}] \cdot \{\mathbf{I}_s\}$$

and we have a the system of linear equations to solve as follows:

$$[\mathbf{M}] \cdot [\mathbf{Z}_b] \cdot [\mathbf{M}^t] \cdot \{\mathbf{I}_m\} = [\mathbf{Z}_m] \cdot \{\mathbf{I}_m\} = -[\mathbf{M}] \cdot [\mathbf{Mu}] \cdot \{\mathbf{I}_s\}$$

We focus on the computed eddy current distribution and loss in the disk (Fig.3 and Tab.1).

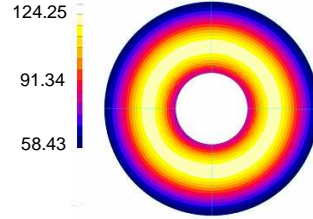


Fig.3. Eddy current distribution in the disk considered ($\delta/\Delta R \approx 0.2$)

The new formulation has been modelled in the case of a skin depth such as $\delta/\Delta R \approx 0.2$. The accuracy is very similar to FEM 3D and the solution has a small relative error compared to the FEM 2D.

Moreover, the convergence of the results is quickly reached with the new formulation. We can obtain it with a smaller number of elements in the disk than the FEM 3D.

TABLE I
RELATIVE ERROR OF OUR FORMULATIONS COMPARED TO THE FEM SOLUTION

Method	$\delta/\Delta R \approx 0.2$	
	Joules Loss(W)	Diff. (%)
FEM 2D	5.35E-7	Ref.
FEM 3D with 1.400.000 elements	5.38E-7	0.69
New formulation with 3000 elements	5.38E-7	0.69

IV. CONCLUSION

In this paper, we have presented an eddy currents volume integral formulation using facet elements in nonmagnetic conductors. The formulation produces very similar results in the comparison with the FEM and allows the treatment of multiply connected domains. In the future, we can improve this formulation by using matrix compression techniques in order to save memory and to reduce the computation time. Moreover, we will to improve the accuracy by using analytical correction to treat the singularity of the mutual inductance terms.

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