Electromagnetic Wave Propagation Simulation in Complex Shaped Domain using Hybrid Method of FDTD and MTDM

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***Abstract*—To simulate electromagnetic wave propagation in complex shaped domain at a reasonable computational cost, a hybrid method of the Finite-Difference Time-Domain method (FDTD) and the Meshless Time-Domain Method (MTDM) has been proposed. In the hybrid method, FDTD and MTDM are chosen according to the shape of domains, i.e., FDTD is used for rectangle domains, and MTDM is used for other kinds of domains. Numerical experiments show that, by using the hybrid method, an electric field is smoothly distributed in a complex shaped domain, including a connection part. In addition, the simulation by using the hybrid method is more efficient than that by using only MTDM in whole domain.**

***Index Terms*—FDTD methods, Electromagnetic propagation, Waveguide bends, Helical waveguides, Maxwell equations**

I. Introduction

In the Large Helical Device (LHD), the Electron Cyclotron Heating (ECH) system is used for plasma heating. In addition, the electrical power that is generated by the gyrotron system transmits to the LHD by using a long corrugated waveguide. However, the shape of curvature of the waveguide or the theoretical transmission gain of electromagnetic wave propagation is not clear.

The Finite-Difference Time-Domain method (FDTD) has generally been applied for electromagnetic wave propagation simulations, and has produced many attractive results [1], [2]. In numerical simulations by using FDTD, the numerical domain has to be divided into rectangle meshes. However, it is difficult that an arbitrary-shaped domain is accurately represented by rectangle meshes.

Recently, the meshless method based on the Radial Point Interpolation Method (RPIM) [3] has been applied to electromagnetic wave propagation simulations [4]. This method is called here the Meshless Time-Domain Method (MTDM). In MTDM, a domain is discretized by using the shape functions of RPIM. Namely, MTDM does not require the rectangle meshes. Hence, the node alignment of MTDM is more flexible than that of FDTD. By using MTDM, electromagnetic wave propagation can be simulated in complex shaped domain such as helical waveguide bends. However, for the case where the number of nodes are same in both methods, the computational cost of MTDM is larger than that of FDTD. If both methods are combined, the combined method may be applied to complex domains at a reasonable computational cost.

The purpose of the present study is to propose a hybrid method of FDTD and MTDM for electromagnetic wave propagation simulations in complex shaped domains.

II. Meshless Time-Domain Method

To simulate electromagnetic wave propagation, we consider Maxwell equations in case of the 2D TM mode described as

$$\varepsilon \frac{\partial E_z}{\partial t} = -\sigma E_z + \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}, \tag{1}$$

$$\mu \frac{\partial H_x}{\partial t} = -\frac{\partial E_z}{\partial y}, \tag{2}$$

$$\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}, \tag{3}$$

where E_z denotes the z component of the electric field, and H_x and H_y denote the x and y components of the magnetic field, respectively. In addition, ε, σ and μ denote the permittivity, the electrical conductivity and the magnetic permeability, respectively.

To discretize (1), (2) and (3) by MTDM, nodes $\boldsymbol{x}_i^E$ ($i = 1, 2, \ldots, N_E$) for E_z and $\boldsymbol{x}_i^H$ ($i = 1, 2, \ldots, N_H$) for H_x and H_y are first aligned in a domain, where N_E denotes the number of nodes for E_z, and N_H denotes the number of nodes for H_x and H_y. In MTDM, the leap-frog method is employed to discretize the time-domain. In addition, the space domain is discretized by using the shape functions of the RPIM. The discretized forms of (1), (2) and (3) are as follows:

$$E_{z,i}^{n} = \frac{\left(\dfrac{\varepsilon}{\Delta t} - \dfrac{\sigma}{2}\right) E_{z,i}^{n-1} + \displaystyle\sum_{j=1}^{N_H} \left(H_{y,j}^{n-\frac{1}{2}} \frac{\partial \phi_{j,i}^{H}}{\partial x} - H_{x,j}^{n-\frac{1}{2}} \frac{\partial \phi_{j,i}^{H}}{\partial y} \right)}{\left(\dfrac{\varepsilon}{\Delta t} + \dfrac{\sigma}{2}\right)} \tag{4}$$

$$H_{x,i}^{n+\frac{1}{2}} = H_{x,i}^{n-\frac{1}{2}} - \frac{\Delta t}{\mu} \sum_{j=1}^{N_E} E_{z,j}^{n} \frac{\partial \phi_{j,i}^{E}}{\partial y}, \tag{5}$$

$$H_{y,i}^{n+\frac{1}{2}} = H_{y,i}^{n-\frac{1}{2}} + \frac{\Delta t}{\mu} \sum_{j=1}^{N_E} E_{z,j}^{n} \frac{\partial \phi_{j,i}^{E}}{\partial x}, \tag{6}$$

where n is the time step, $E_{z,i}^{n} \equiv E_z^n(\boldsymbol{x}_i^E)$, $H_{x,i}^{n+\frac{1}{2}} \equiv H_x^{n+\frac{1}{2}}(\boldsymbol{x}_i^H)$, and $H_{y,i}^{n+\frac{1}{2}} \equiv H_y^{n+\frac{1}{2}}(\boldsymbol{x}_i^H)$. In addition, $\phi_j^E(\boldsymbol{x})$ denotes the shape functions corresponding to $\boldsymbol{x}_j^E$ ($j = 1, 2, \ldots, N_E$). Similarly,

$\phi_j^H(\boldsymbol{x})$ denotes the shape functions corresponding to $\boldsymbol{x}_j^H$ ($j = 1, 2, \ldots, N_H$). Note that the shape functions $\phi_j^E(\boldsymbol{x})$ and $\phi_j^H(\boldsymbol{x})$ have the Kronecker delta function property [3].

III. Hybrid Method of FDTD and MTDM

In FDTD, a number of nodes have to be generated to represent complex shaped domains, since rectangle meshes are generally employed in FDTD. Thus, in electromagnetic wave propagation simulations using FDTD in complex shaped domains, the computational cost tends to be large.

On the other hand, the node alignment of MTDM is more flexible than that of FDTD. Hence, in electromagnetic wave propagation simulations in complex shaped domains, the number of nodes for MTDM is smaller than that for FDTD. However, in simple shaped domains such as line waveguides, the computational cost of MTDM is larger than that of FDTD for the case where the number of nodes are same in both methods.

In this section, we describe a hybrid method of FDTD and MTDM to simulate electromagnetic wave propagation in complex shaped domains at a reasonable computational cost. In the hybrid method, FDTD and MTDM are chosen according to the shape of domains, i.e., FDTD is used for rectangle domains, and MTDM is used for other kinds of domains.

In the connection part between the domain Ω_{F} of FDTD and the domain Ω_{M} of MTDM, some strategies may be considered for natural propagation. In this study, we consider some overlap between Ω_{F} and Ω_{M}. In the overlapping domain, $E_z^n, H_x^{n+\frac{1}{2}}$ and $H_y^{n+\frac{1}{2}}$ are calculated by both methods. In addition, $E_z^n, H_x^{n+\frac{1}{2}}$ and $H_y^{n+\frac{1}{2}}$ are simply determined as the average of values calculated by both methods.

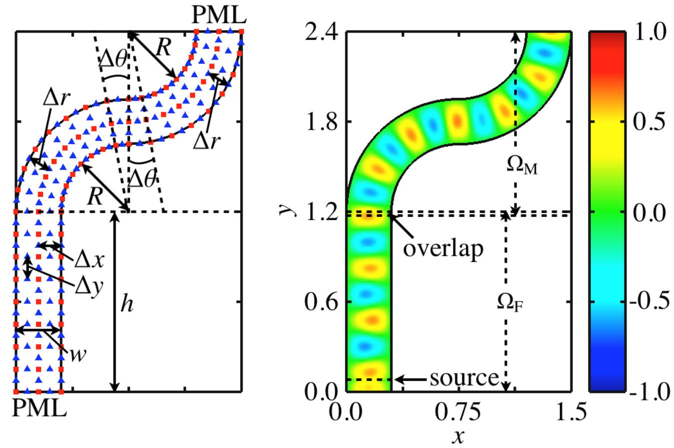
IV. Numerical Experiments

In this section, numerical experiments are conducted to investigate a performance of the hybrid method of FDTD and MTDM for a 2D electromagnetic wave propagation simulation in a complex shaped domain. To this end, a waveguide bend illustrated in Fig. 1(a) is used for this simulation. In addition, the nodes $\boldsymbol{x}_i^E$ and $\boldsymbol{x}_i^H$ are uniformly aligned like the staggered mesh as shown in Fig. 1(a). Throughout this section, the parameters for the simulation are fixed as shown in Table I.

To generate shape functions $\phi_i^E(\boldsymbol{x})$ and $\phi_i^H(\boldsymbol{x})$ for MTDM, the exponential weight function [5],

$$w_i(r) \equiv \begin{cases} \dfrac{\exp[-(r/c)^2] - \exp[-(R_i/c)^2]}{1 - \exp[-(R_i/c)^2]} & (r \leq R_i), \\ 0 & (r > R_i), \end{cases} \tag{7}$$

is adopted (see [3] for details of generating shape functions of RPIM). Here, $r \equiv |\boldsymbol{x}_i - \boldsymbol{x}|$, R_i denotes a support radius of $w_i(r)$, and c is a user-specified parameter. We set $R_i = 3d$ and $c = d$, where d is the average of the minimum distance between two nodes. In addition, we set $R_E = d^E$ and $R_H = d^H$, where R_E and R_H are support radii of shape functions $\phi_i^E(\boldsymbol{x})$ and $\phi_i^H(\boldsymbol{x})$, respectively, and d^E and d^H denote d of $\boldsymbol{x}_i^E$ and that of $\boldsymbol{x}_i^H$, respectively.



(a) (b)

Figure 1: (a) Schematic view of a waveguide for experiments. Node alignment of $\boldsymbol{x}^E$ and that of $\boldsymbol{x}^H$ are represented as red quadrilaterals and blue triangles, respectively. Here, $w = 0.3$m, $h = 1.2$m, $R = 0.45$m. (b) Distribution of the electric field E_z obtained by the hybrid method of FDTD and MTDM.

Table I: Parameters for the numerical experiments.

Wave source	Sine wave
Amplitude	1.0 V/m
Frequency	1.0×10^9 Hz
Wave speed	299792458 m/s
Number of layer for PML	16
Reflectivity coefficient of PML	-120 dB
Δx, Δy and Δr	0.015 m
$\Delta\theta$	$\pi/40$

The domains Ω_{F} and Ω_{M} are set as shown in Fig. 1(b). Note that the width of the overlapping domain between Ω_{F} and Ω_{M} is Δy. The distribution of E_z that determined by the hybrid method is shown in Fig. 1(b). We see from this figure that E_z is smoothly distributed, including the overlapping domain. Note that the simulation by using the hybrid method is more efficient than that by using only MTDM in whole domain. This is because the shape functions have to be generated in MTDM. In the hybrid method, the number of shape functions for MTDM is decreased obviously, since FDTD is employed for rectangle domains. Hence, we consider that, by using the hybrid method, the electromagnetic wave propagation can be simulated in complex shaped domains at a reasonable computational cost.

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