A New Basis Function for Fast Computation of Electromagnetic Fields in Meshless Frames

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***Abstract*—In this study, we try to develop the application of meshless method, particularly conventional radial point interpolation method (CRPIM), in computational electromagnetics. Indeed, this development focuses on the two important factors of every numerical algorithm, i.e., the computational time and the accuracy. Comparing the identity of shape and basis functions in RPIM, a new algorithm is proposed by which the shape and basis functions coincide. As a result, the computational time of existing RPIM in electromagnetics reduces, approximately to half. Also, the accuracy improves even in comparison with some developed methods as finite element method (FEM) and CRPIM. All improvements are shown in analysis of a parallel plate waveguide with internal discontinuity.**

***Index Terms*—Meshless Method, functional analysis, basis function, shape function, parallel waveguide.**

I. Introduction

The RPIM is one of the most common and efficient meshless techniques in computational electromagnetics. Due to the independency of this method from any mesh generation, potentially, it is applicable to more complicated geometries. Also, in contrast with FEM whose approximation spaces are of polynomials, the RPIM uses variety kinds of approximation spaces as exponential, logarithmic and etc [1]. So, its accuracy can be more than FEM in some complicated wave propagations as discontinuity interfaces [2].

The computational time of RPIM is the main disadvantage of the method, comparing with some other well-established techniques. Above problem becomes more inefficient when the RPIM is used as an industrial numerical package. The source of this problem is the middle inversion matrix step (MIMS) that constructs the shape functions according to basis functions [3].

In this work, using the property of exponential basis function, a new basis function is proposed by which the MIMS is eliminated. In fact, the new basis function takes a special form such that appears as shape function. This new function reduces the computational time of the meshless method. Furthermore, it increases the accuracy of RPIM. This increment is due to the elimination of some numerical error sources related to MIMS.

To see the improvements, practically, a parallel plate waveguide with an internal dielectric that acts as discontinuity will be analyzed using RPIM supplemented by the new basis function. Both error calculation and computational time of this simulation show the validity of approach.

II. The Conventional Radial Point Interpolation Method

Let consider the general form of 2D scalar wave equation as

$$\frac{\partial}{\partial x}\left(a_x\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(a_y\frac{\partial u}{\partial y}\right)-gu=-f, \quad (x,y)\in\Omega \tag{1}$$

where Ω is the problem domain, a_x, a_y, g and f are known functions, supplemented by the following Dirichlet and mixed boundary conditions as

$$u=b_0, \quad \textit{on curve } L_1 \tag{2}$$

$$\left(a_x\frac{\partial u}{\partial x}\hat{i}+a_y\frac{\partial u}{\partial y}\hat{j}\right).\hat{n}+\gamma u=q, \quad \textit{on curve } L_2 \tag{3}$$

where $L = L_1 + L_2$ is the counter, enclosing Ω with outward normal vector $\hat{n}$. Again, γ, q and b_0 are known functions [4].

Using Ritz's method, the functional (weak form) of above equations is

$$\begin{aligned} F(u) &= \frac{1}{2}\int_\Omega\left[a_x\left(\frac{\partial u}{\partial x}\right)^2+a_y\left(\frac{\partial u}{\partial y}\right)^2+gu^2-2fu\right]d\Omega \\ &+ \int_{L_2}\left(\frac{\gamma}{2}u^2-qu\right)dL_2 \end{aligned} \tag{4}$$

The CRPIM proposes the following approximation as the solution of (4)

$$\tilde{u}(x,y)=\tilde{u}=\sum_{i=1}^{n}a_iB_i(x,y) \tag{5}$$

where a_i is unknown coefficient and $B_i(x,y)$ is the *basis function*, both at ith node; n is the number of scattered nodes in the problem domain Ω [2]. The matrix representation of (5) is

$$\tilde{u}=AB^T \tag{6}$$

A. Conventional Basis Function

One of the most common basis functions in CRPIM is the exponential one by shape parameter α, as

$$B_i(x,y)=exp\left(-\alpha\left[(x-x_i)^2+(y-y_i)^2\right]\right) \tag{7}$$

Even though (5) is a solution, but it must be rewritten according to the value of solution function at scattered nodes [3]. So, the CRPIM proposes the following approximation, equivalent to (5), as

$$\tilde{u}=\sum_{i=1}^{n}u_iS_i(x,y) \tag{8}$$

where u_i is the value of solution function and S_i is the *shape function*, both at ith node [1]. The matrix representation of above equation is

$$\tilde{u} = US^T \tag{9}$$

To find the shape functions, a $(n \times n)$ MIMS is constructed as below

$$S^T = B^T B_0^{-1} \tag{10}$$

where

$$\mathbf{B_0} = \begin{pmatrix} B_1(x_1, y_1) & \dots & B_n(x_1, y_1) \\ B_1(x_2, y_2) & \dots & B_n(x_2, y_2) \\ \vdots & \ddots & \vdots \\ B_1(x_n, y_n) & \dots & B_n(x_n, y_n) \end{pmatrix} \tag{11}$$

Afterwards, the system of equations is assembled substituting (8) into (4) as

$$[K_{ij}]_{n\times n} U_{n\times 1} = [b_{i1}]_{n\times 1} \tag{12}$$

Both (10) and (12) take noticeable computational time in calculation process.

B. New Basis Function

When S_i is derived, it possesses the three basic properties, i.e., compactly supported, even symmetry and Kronecker delta property [1] as shown in Figure 1. For the sake of generality (and due to the strict page limitation of short paper submission), let express the new (direct) basis function, directly, as

$$\begin{aligned} B_i^{direct}(x, y) &= \left(-\beta\left[(x - x_i)^2 + (y - y_i)^2\right] + 1\right) \\ &\times \exp\left(-\alpha\left[(x - x_i)^2 + (y - y_i)^2\right]\right) \end{aligned} \tag{13}$$

that contains two shape parameters α and β to control its shape. More attention illustrates the role of α and β to take those values by which all three properties of shape functions are also satisfied by B_i^{direct} [5]. Using trial and error method, $\alpha = 1.92$ and $\beta = 0.97$ realize the aim (more concentration is on Kronecker delta property).

Holding above properties, (11) changes to

$$B_0^{direct} = I \tag{14}$$

as identity matrix and finally

$$S = B^{direct} \tag{15}$$

Indeed, this approach cancels the MIMS, reduces the computational time and increases the accuracy of direct RPIM (DRPIM).

III. Parallel Plate Waveguide with Internal Discontinuity

To prove the improvements in practice, the DRPIM is applied to analyze a 2D parallel plate waveguide (Figure 2) in which a partition of dielectric with $\varepsilon_r = 4$ acts as discontinuity zone. Without any mention to governing equations (they can be found in [4]), both reflection and transmission coefficients of above structure has been calculated using different methods. Figure 3 shows the error plots of these methods As seen, the DRPIM is able to reach a given error in less computational time and number of nodes.

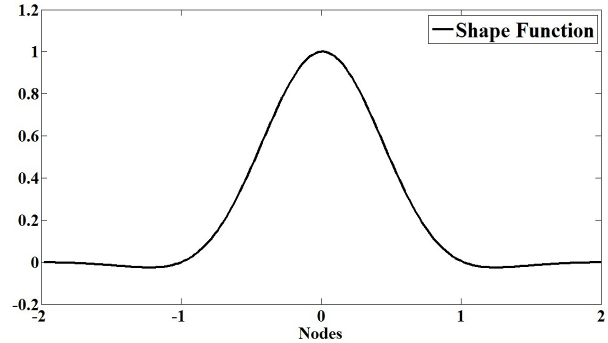

Figure 1: Proposed shape (direct basis) function at $x_i = y_i = 0$.

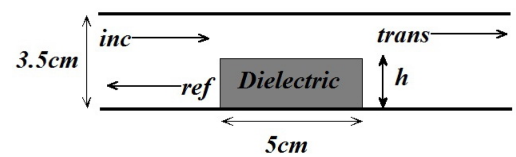

Figure 2: Parallel plate waveguide with discontinuity.

IV. Conclusion

Identifying the main problem of CRPIM, i.e., computational time, this paper proposed a new basis function that is able to coincide with shape functions and cancel the MIMS.

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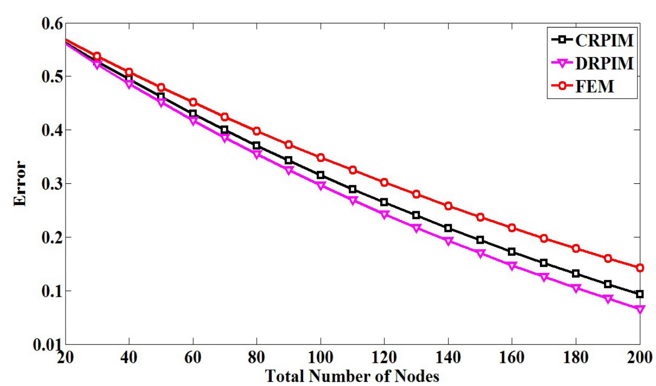

Figure 3: 2D error analysis using different methods.