An Accurate Multi-layer Magnetic Force Computation Method by Using Adaptive Parameterized Mesh Technique

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***Abstract*—How to compute the magnetic force of an object placed in magnetic field using finite element method (FEM) precisely is an important problem in electrical engineering. In this paper a multi-layer fore computation method is combined with a parameterized mesh technique to get reliable and accurate magnetic force results. The multi-layer force calculation method is to average the force value obtained on each layer surrounding the object whose force is analyzed to improve the result. This idea is extended using the parameterized mesh technique. By constructing several virtual parameterized layers around the considered object, the layer-averaged force for each parameter is further averaged versus the number of parameters to improve the force result again. The accuracy of this parameterized mesh multi-layer method is very promising and satisfactory. Numerical examples are reported to validate the accuracy of this proposed method.**

***Index Terms*—Adaptive mesh refinement, finite element analysis, force calculation, magnetic field, parameterized mesh.**

I. Introduction

The finite element method is widely used in magnetic field computation which has been proved to be a powerful tool for the analysis and design of electromagnetic devices. In many applications, the magnetic force acting on an object in the magnetic field is an important physical quantity to know. Force computation is fundamental and essential for analyzing the performance of electromagnetic (EM) devices [1].

After solving the magnetic field by the FEM, the force can be obtained from the finite element (FE) solution by post-processing. There are basically two formulations for the force calculation according to the field solution. One is based on the Maxwell stress tensor, and the other is based on the physical principle of virtual work [2].

To improve the accuracy of the FE solution and hence the derived magnetic force result, uniformly refined meshes are too expensive to use and the locally refined adaptive mesh if of significant importance in practice. The ideal FE mesh should be composed of good-shaped elements and the nodes should be dense enough and properly distributed, which can be fine-tuned by the adaptive mesh method.

In this paper, the adaptive mesh refinement method is used to get accurate FE solution with minimal computational resources. A multi-layer force computation method is proposed by averaging the forces for each layer in order to produce more accurate magnetic forces than the traditional one-layer method. This idea is further extended by constructing several virtual parameterized layers around the considered object, where the layer-averaged forces for each parameter are averaged versus the number of parameters to improve the force result again. The accuracy of this parameterized mesh multi-layer method is very promising and satisfactory. Numerical examples are reported to validate the accuracy and usefulness of this proposed method.

II. Proposed Force Calculation Method

The procedures of the proposed force calculation method composed of several steps:

(1) Construct a parameterized geometry of the considered device with several virtual layers around the object whose force is to be analyzed, as shown in Fig. 1. Construct the initial mesh and adaptively refine it several times for FE computation.

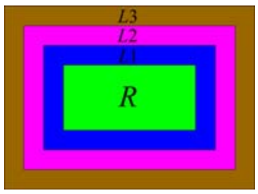


Fig. 1. Multi-layer force computation method, where the inner-most rectangle R is the object whose force is to be computed, the three outer layers $\{L_1, L_2, L_3\}$ are virtual objects whose forces are to be averaged.

(2) Select a parameter, perform FE computation with the parameterized mesh and calculate the magnetic force using the multi-layer force calculation method with the virtual work formulation, the force calculated for this parameter is stored.

(3) Change the value of the parameter and hence the mesh in the layers and calculate the force on each layer again using the multi-layer force computation method.

(4) Average all the force values for each parameter to get the final magnetic force result.

A. *Parameterized Mesh Technique*

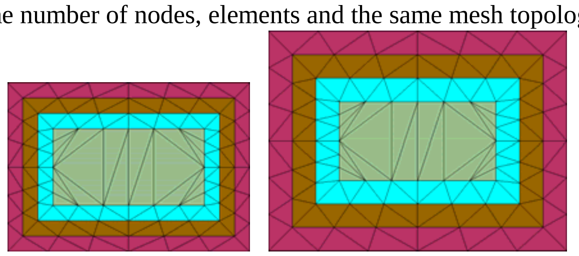
In the proposed parameterized mesh multi-layer force calculation method, several virtual objects surrounding the object whose force is to be analyzed are constructed with their sizes depending on the parameters. Then these parameterized layers are deformed several times without the need to generate the new mesh again. For different layer parameters, the magnetic fields are solved using the FEM and the forces are calculated by the multi-layer method. Then the layer-averaged force for each parameter is further averaged versus the number of parameters to improve the force result again.

In the parameterized mesh method [1,3], when the parameters change, the new mesh does not need to be generated repeatedly. Take a two dimensional (2-D) triangular mesh for example. The key point is to write the coordinates of a node i in a FE mesh as functions of the parameters p_1, p_2, …, p_N:

$$\begin{Bmatrix} x_i \\ y_i \end{Bmatrix} = \begin{bmatrix} C_{ix0} & C_{ix1} & C_{ix2} & \cdots & C_{ixN} \\ C_{iy0} & C_{iy1} & C_{iy2} & \cdots & C_{iyN} \end{bmatrix} \begin{Bmatrix} 1 \\ p_1 \\ p_2 \\ \vdots \\ p_N \end{Bmatrix} = \begin{bmatrix} C_{ix} \\ C_{iy} \end{bmatrix} \begin{Bmatrix} 1 \\ p \end{Bmatrix}, \qquad (1)$$

where p_1, p_2, …, p_N are geometric parameters, $[C_{ix}]$ and $[C_{iy}]$ are the coefficient matrices. By (1), each node in the mesh is assigned a set of weights under the parameters. It is easy to see that when $\{p\}$ varies, the coordinates of all nodes in the mesh will be changed accordingly and no remeshing is needed to make the new mesh.

For example, the original mesh and the deformed mesh using the parameterized mesh method are shown in Figs. 2(a)-2(b), where one can observe that the two meshes are with the same number of nodes, elements and the same mesh topology.



(a) (b)

Fig. 2. Original mesh and the new updated mesh mesh by resetting the coordinates using the parameterized mesh method.

B. Force Computation by Virtual Work Principle

The total force for the considered object can be calculated by the virtual work method as given by [2]

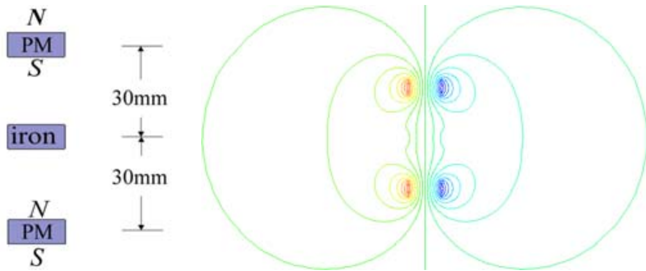
$$F = l_z \sum_e \int_{\hat{\Omega}_e} \left[-B^T J^{-1} \frac{\partial J}{\partial s} H + \int_0^H B dH |J|^{-1} \frac{\partial |J|}{\partial s} \right] d\hat{\Omega}, \qquad (2)$$

where s is a virtual displacement vector in the generalized coordinate system, J is Jacobian matrix and $|J|$ is the determinant of it, l_z is the length in the z-direction of the object. Note that the standard virtual work method, the relevant FE solution is integrated along only one layer of elements $\{e\}$ that are surrounding the object analyzed.

III. Numerical Example

The first example is to calculate the magnetic force acting on an infinitely long iron bar (with relative permeability $\mu_r = 4000$) placed in the middle of two parallel infinitely long permanent magnets (PMs). Since this example is a symmetric problem and the exact total force of the iron bar should be zero.

The cross sections of all the three bars are all of size of $10\text{mm} \times 20\text{mm}$, the distances between the two PMs and the iron bar are both 30 mm, as shown in Fig. 3(a). The magnetization direction of the two PMs are set to be along the y axis with B_r = 1.1 tesla. The solution contour lines are shown in Fig. 3(b).



(a) (b)

Fig. 3. (a) Problem setting for the example 1. (b) The solution contour lines.

TABLE I

FORCE RESULTS FOR EXAMPLE I WITH THREE LAYERS

Layer thickness (mm)	Average F_x (N)	Average F_y (N)
1.0	-0.0014	-0.0084
1.5	0.0005	0.0046
2.0	0.0024	-0.0095
2.5	0.0046	0.011
3.0	-0.0004	-0.0019
3.5	0.0016	0.0011
4.0	-0.0013	-0.002
4.5	-0.0039	-0.082
5.0	0.0003	-0.0073
5.5	0.0003	0.012
6.0	0.0002	0.0007
Final force result	-0.00022	-0.000052

The accuracy of the proposed method is obvious. The force value calculated by the parameterized mesh multi-layer force calculation method is more accurate than that obtained by using multi-layer force calculation method only, as can be seen in Table I. Since the analytical result is 0 for both the x-component and the y-component of the total force, the final force result with F_x = -0.00022N and F_y = -0.000052N is very satisfactory. More complicated examples and the three-dimensional examples are presented in the full paper.

References

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