Solution of Large Complex BEM Systems Derived from High-Resolution Human Models

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***Abstract*—This paper discusses some numerical properties of the Boundary Element matrices derived from the electromagnetic analysis of high resolution human databases. Different solutions related to the choice of the problem equations and of the solver are proposed and compared by application to test problems.**

***Index Terms*—Boundary element methods, Electromagnetic fields, Matrix inversion.**

I. Introduction

The evaluation of human exposure to electromagnetic radiating sources requires a detailed model of the body with the tissue properties and an efficient numerical method for the field solution in highly heterogeneous domains. A combined use of a voxel-based anatomic model [e.g. 1, 2] with the Boundary Element Method (BEM) [3] is a promising approach for this application. However in frequency domain this technique leads to complex systems whose matrices have very large numbers of non zero entries per row, so needing very robust and efficient solvers.

This paper investigates from a numerical point of view the electromagnetic BEM analysis applied to high-resolution human models. Particular attention is devoted to the way of combining the field equations internal and external with respect to the surfaces bounding the volumes into which the body is divided, because this choice strongly affects the properties of the associated matrix. Then, different sparse direct and iterative solvers are applied for the system solution. Both open source direct solvers (UMFPACK [4], SuperLU [5] and SPOOLES [6]), and iterative solvers implemented by the authors are used. The reliability and accuracy of the numerical solution are finally discussed in some applications.

II. Field Formulation

The electromagnetic field problem is described by the Electric Field Integral Equation (EFIE) and Magnetic Field Integral Equation (MFIE), where, under sinusoidal conditions (angular frequency ω), the Green's function is given by:

$$\psi = \frac{e^{-ik|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|} \quad \text{with } k=\omega\sqrt{\mu\left(\varepsilon - j\frac{\sigma}{\omega}\right)}=\omega\sqrt{\mu\tilde{\varepsilon}}$$

where $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the coordinate vectors of the observation and of the source points, while μ, σ and ε are the magnetic permeability, the electric conductivity and the electric permittivity respectively.

The body is divided into homogeneous volumes generated by joining together adjacent voxels belonging to the same tissue. The resulting bounding surfaces of each volume are constituted by quadrangular elements (voxel faces).The electric and magnetic fields, assumed to be uniform on each element, are the unknowns. Thus, for a sub-volume Ω, bounded by M quadrangles, the field problem is governed by:

$$\xi\boldsymbol{E}_i = \int_{\Omega_s}\left(\frac{\rho}{\varepsilon}\nabla\Psi - j\omega\mu\Psi\boldsymbol{J}_s\right)dv - \sum_m^M(\boldsymbol{n}\times\boldsymbol{E})_m\times\int_{\partial\Omega_m}\nabla\Psi_{i,m}ds - \sum_m^M(\boldsymbol{n}\cdot\boldsymbol{E})_m\int_{\partial\Omega_m}\nabla\Psi_{i,m}ds + j\omega\mu\sum_m^M(\boldsymbol{n}\times\boldsymbol{H})_m\int_{\partial\Omega_m}\Psi_{i,m}ds$$

$$\xi\boldsymbol{H}_i = \int_{\Omega_s}(\boldsymbol{J}_s\times\nabla\Psi)dv - \sum_m^M(\boldsymbol{n}\times\boldsymbol{H})_m\times\int_{\partial\Omega_m}\nabla\Psi_{i,m}ds - \sum_m^M(\boldsymbol{n}\cdot\boldsymbol{H})_m\int_{\partial\Omega_m}\nabla\Psi_{i,m}ds - j\omega\tilde{\varepsilon}\sum_m^M(\boldsymbol{n}\times\boldsymbol{E})_m\int_{\partial\Omega_m}\Psi_{i,m}ds$$

where $\boldsymbol{J}_s$ and ρ are the impressed current density and the volume charge density of the sources (typically outside the body), ξ is the singularity factor (ξ = 0.5 on the surface and ξ = 1 elsewhere) and $\boldsymbol{n}$ is the normal unit vector directed outwards Ω. The m-th element is the source point while, during the setting of the matrix, the computational point is the barycentre of the i-th element. Each vectorial equation is then transformed in a set of three scalar relations by projecting it on the normal and on two tangential unit vectors defined for the i-th element.

For each quadrangle the problem involves six complex unknowns (the components of both the electric and magnetic fields), that can be defined on one side (conventionally called internal) or on the other one (external). We always adopt as unknowns the normal component of $\boldsymbol{E}$ and the tangential components of $\boldsymbol{H}$ on the "internal" side, while the tangential components of $\boldsymbol{E}$ and the normal component of $\boldsymbol{H}$ are assumed on the "external" side. Obviously, internal and external quantities can be linked through the interface conditions.

III. Application and Numerical Analysis

The scalar equations associated to each unknown could be theoretically written with reference to the internal or external side. The case where all the relations are written on the same side (internal or external) is excluded, because it leads to a trivial problem. Three of the other possible choices to mix the two points of view are here investigated:

A1) The relations are written according to the choice of the unknowns (e.g. the equation for the normal component of $\boldsymbol{E}$ is developed in the inner volume, the relation for the normal component of $\boldsymbol{H}$ refers to the external region and so on).

A2) For each unknown the couples of equations written on the internal and external side are summed together.

A3) The equations are written as in the previous case, but then they are combined with suitable complex weights so that the main diagonal of the matrix becomes unitary.

The proposed approach is applied to a portion of a human model belonging to the Virtual Family [4], radiated by a coil which is supplied at 100 kHz and 100 MHz. A first representation of this domain leads to a problem with 1626 unknowns and 1132714 non-zero matrix elements. The analysis of the three resulting BEM matrices for the two frequencies is summarized in Tab. I, which presents the inverse conditioning number for all the matrices. The best quality of choice A2 is evident.

TABLE I
INVERSE CONDITIONING NUMBER

100 kHz			100 MHz		
A1	A2	A3	A1	A2	A3
8.3E-6	5.5E-5	6.9E-12	4.7E-5	6.2E-4	2.2E-4

Both sparse direct and iterative solvers are considered in the system solution. The direct solvers are more robust than the iterative ones, being capable of computing a solution almost independently from the matrix properties; however, they require more memory and CPU resources. Moreover, since implementing a sparse direct solver is a very challenging task, it is common practice to use already available sparse direct solvers, both closed source and open source, rather than coding them. It is to be noted that most of the existing sparse direct solvers were developed for finite difference or finite element matrices, i.e. for matrices much more sparse than the BEM matrices. The sparse direct solvers here compared are UMFPACK [5], SuperLU [6] and SPOOLES [7], indicated as M1, M2, M3, respectively; however, only SPOOLES is capable of performing its task on the BEM matrices.

The Lanczos family iterative solvers [8] are the most popular methods in computational electromagnetics. Between them, only GMRES (M4) and BiCGSTAB can be chosen because the systems to be solved have complex and non-symmetric matrices, while older solvers (BiCG and CGS) are not considered here. Every Lanczos-based method performs matrix-vector products and auxiliary operations on the product results. Generally, the solver efficiency depends on the balancing of the amount of matrix-vector products with the auxiliary operations. In this regard, GMRES minimizes the number of matrix-vector products at the cost of heavy auxiliary operations, while BiCGSTAB requires much lighter auxiliary operations but a higher amount of matrix-vector products. For the BEM matrices, having many non zero entries per row, it is preferable to minimize the amount of matrix-vector products, hence GMRES is expected to be the best performing.

The performances of four solvers, applied to the previous domain, are compared in terms of expecting time (s) and presented in Tab. II, where for GMRES (restarting parameter p = 20, convergence tolerance = $1\cdot10^{-6}$) the iteration number is reported between brakets. The superiority of GMRES is not a matter of discussion and the best quality of combination A2 is also confirmed.

TABLE II
EXPECTING TIME (s) FOR THE CONSIDERED SOLVERS

	100 kHz			100 MHz		
	A1	A2	A3	A1	A2	A3
M1	Fail	Fail	Fail	Fail	Fail	Fail
M2	3.30	3.30	3.34	3.31	3.32	3.45
M3	1.92	1.91	29.14	1.93	1.94	1.93
M4	0.68 (72)	0.37 (13)	-	0.97 (125)	0.27 (21)	0.53 (45)

In a more detailed domain model (36804 unknowns; non-232963060 zero elements) a small cylinder, simulating a medical implant, is included. In Tab. III the comparison (in terms of expecting time and number of iterations) is performed only for arrangement A2 at a frequency of 100 MHz, varying the cylinder conductivity (σ_1 = 1 MS/m; σ_2 = 0.5 MS/m; σ_3 = 0.1 MS/m;). The SuperLU (M2) always fails, reporting not enough memory to perform factorization. In GMRES the convergence tolerance is fixed to $1\cdot10^{-6}$ and some values of the restarting parameter p are analyzed.

TABLE III
EXPECTING TIME (s) FOR THE CONSIDERED SOLVERS

	p	σ_1	σ_2	σ_3
M3	-	4074.	4090.	4099.
M4	20	Fail	Fail	1313. (2362)
M4	40	685. (1227)	546. (980)	248. (442)
M4	60	375. (671)	338. (600)	168. (303)
M4	80	235. (423)	182. (327)	122. (218)
M4	100	195. (351)	166. (297)	119. (215)

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