Thin Conducting Sheet (TCS) in non-destructive testing simulations: implementation in Code_Carmel3D and validation

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***Abstract*—This paper presents a novel thin conducting sheet (TCS) model. This model is used to replace 3D thin conductive domains in eddy current simulations by an equivalent impedance boundary condition coupling the tangential electromagnetic fields on both sides of the sheet. It has been implemented in $A-\Phi$ and $T-\Omega$ finite element formulations of Code_Carmel3D. This approach is validated by an analytic comparison (spherical shield). In this paper, it is used to model a thin conductive layer deposited on the outside surface of a tube.**

***Index Terms*—eddy currents, finite element method, surface impedance, electromagnetics**

I. Introduction

In nuclear power plants, several Non-Destructive Testing (NDT) techniques are used to check the tubes of steam generators. Inspection by eddy-current probes is one of these NDT techniques. Finite Element (FEM) simulations are widely used to improve the inspection procedures, or to evaluate the probe ability to detect flaws.

The accurate modelling of conductive domains with a small skin depth (such as the Tube Support Plate) is classically achieved by replacing the 3D domains by impedance boundary conditions [1], [5].

Another severe mesh issue is raised by thin conductive layer deposited on tubes. The difficulty is not (only) related to the skin depth anymore, but to the geometrical size of the layer. Thin Conducting Sheet (TCS) models have proved to be very efficient in FDTD simulations [3]. So we propose to couple the FEM computation to a Thin Conducting Sheet (TCS) model to overcome this issue.

We first state the TCS model and its implementation in Code_Carmel3D[1] finite element formulations $(\vec{A},\Phi)$ and $(\vec{T},\Omega)$. We describe its validation and show its efficiency to model a thin deposit on a steam generator tube. We point out that the TCS model allows to tackle a wide range of problems: thin conducting sheets with or without holes, with or without edges, for any value of the skin depth.

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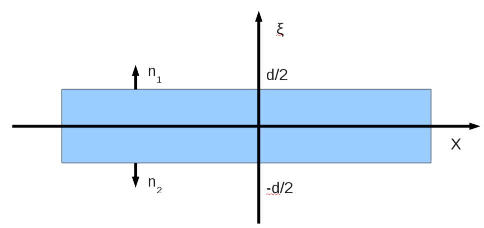

Figure 1: Geometry of the conductive thin sheet Γ_{TCS}.

II. Numerical Method

The electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ satisfy the eddy current harmonic problem :

$$\begin{aligned} \operatorname{curl}\mathbf{E} + i\omega\mu\mathbf{H} &= 0, \\ \operatorname{curl}\mathbf{H} - \sigma\mathbf{E} &= \mathbf{J}_s, \end{aligned} \tag{1}$$

where ω is the angular frequency, σ the conductivity and $\mathbf{J}_s$ is a known source current (e.g. the current in the emitting coil of the probe in NDT simulations). The skin depth δ is defined by

$$\delta = \sqrt{2/\sigma\mu\omega}. \tag{2}$$

A. The TCS equivalent model

Let us consider an thin conductive layer, of thickness d (see Fig. 1). We denote by $\mathbf{n} = \nabla\xi = \mathbf{n}^1 = -\mathbf{n}^2$ the normal to the layer.

We make use of the fact that the spatial variation of the fields normal to the surface is much more rapid than the variations parallel to the surface [2] to approximate the gradient operator by

$$\nabla = -\mathbf{n}\partial_\xi.$$

Hence, the fields inside the conductive sheet are parallel to the surface, and obey a 1D equation. The tangential fields $\mathbf{E}_T^1$,

Table I: Validation of the TCS model: spherical conductive shield of center 0. We show the (relative) error ϵ between the analytical value of $\|\mathbf{B}(0)\|$ and the value computed with the TCS model, for both formulations of Code_Carmel3D: electric $(\vec{A}, \Phi)$ and magnetic $(\vec{T}, \Omega)$

f	d/δ	ϵ (%) $(\vec{A}, \Phi)$ formulation	ϵ (%) $(\vec{T}, \Omega)$ formulation
2.5 kHz	0.38476	6.6%	6.6%
250 kHz	3.8476	7.2%	7.2%
4 MHz	15.3905	6.4%	6.4%

$\mathbf{H}_T^1$ (on the upper side of the layer) and $\mathbf{E}_T^2$, $\mathbf{H}_T^2$ (on the lower side) of the layer are linked by the impedance matrix relation:

$$\begin{pmatrix} \mathbf{E}_T^1 \\ -\mathbf{E}_T^2 \end{pmatrix} = \underbrace{\begin{pmatrix} -z_d[I] & z_n[I] \\ z_n[I] & -z_d[I] \end{pmatrix}}_{=Z_{TCS}} \begin{pmatrix} \mathbf{H}_T^1 \wedge \mathbf{n} \\ \mathbf{H}_T^2 \wedge \mathbf{n} \end{pmatrix}, \tag{3}$$

where

$$z_d(\omega) = \frac{k}{\sigma} \frac{\cos(kd)}{\sin(kd)}, \; z_n(\omega) = \frac{k}{\sigma} \frac{1}{\sin(kd)}, \; k^2 = -i\omega\mu\sigma. \tag{4}$$

The impedance relation (3) can also be written as an admittance relation :

$$\begin{pmatrix} \mathbf{H}_T^1 \\ -\mathbf{H}_T^2 \end{pmatrix} = \underbrace{\begin{pmatrix} -y_d[I] & y_n[I] \\ y_n[I] & -y_d[I] \end{pmatrix}}_{=Y_{TCS}} \begin{pmatrix} \mathbf{E}_T^1 \wedge \mathbf{n} \\ \mathbf{E}_T^2 \wedge \mathbf{n} \end{pmatrix}, \tag{5}$$

where

$$y_d(\omega) = \frac{\sigma}{k} \frac{\cos(kd)}{\sin(kd)}, \; y_n(\omega) = \frac{\sigma}{k} \frac{1}{\sin(kd)}. \tag{6}$$

B. Finite Element Formulation

The electric weak formulation of (1) in domain $\mathcal{D}$ is:

$$\left\{ \begin{array}{l} \text{Seek } \mathbf{E} \text{ such that for all test function } \mathbf{E}', \\ \underbrace{-\int_{\mathcal{D}} (i\omega\mu)^{-1} \operatorname{curl} \mathbf{E} \cdot \operatorname{curl} \mathbf{E}' - \int_{\mathcal{D}} \sigma \mathbf{E} \cdot \operatorname{curl} \mathbf{E}'}_{Y_{\mathcal{D}}(\mathbf{E}, \mathbf{E}')} = \int_{\mathcal{D}} \mathbf{J}_s \cdot \mathbf{E}'. \end{array} \right.$$

We then work on the contribution of the TCS domain to Y:

$$\begin{aligned} Y_{\mathcal{D}_{TCS}}(\mathbf{E}, \mathbf{E}') &= -\int_{\partial \mathcal{D}_{TCS}} \mathbf{H} \cdot (\mathbf{E}' \wedge \mathbf{n}) \\ &\approx -\int_{\Gamma_{TCS}} (\mathbf{E}^{1,t} \wedge \mathbf{n}, \mathbf{E}^{2,t} \wedge \mathbf{n})^H [Y_{TCS}] (\mathbf{E}^1 \wedge \mathbf{n}, \mathbf{E}^2 \wedge \mathbf{n}) \, d\Gamma \end{aligned}$$

where the 2D surface Γ_{TCS} replaces the 3D domain $\mathcal{D}_{TCS}$. The final step is to introduce the potentials $(\vec{A}, \Phi)$. A similar computation leads to the $(\vec{T}, \Omega)$ magnetic formulation.

III. Validation

We use an analytical test case : a thin spherical shield in a uniform magnetic field. All results have been run with Code_Carmel3D software, in which the TCS model has been implemented. We consider different frequencies, so as to validate the efficiency of the TCS model for different skin depth ranges: $\delta \ll d$, $\delta \approx d$, $\delta \gg d$. Table I recaps the error between the analytical value of the magnetic induction $\mathbf{B}$ at the center of the sphere, and the numerical value respectively obtained with the $(\vec{A}, \Phi)$ and $(\vec{T}, \Omega)$ formulations.

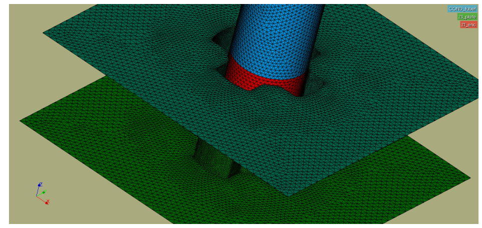

Figure 2: Mesh of the tube (3D mesh, FEM formulation, blue), quadrifoiled Tube Support Plate (2D mesh of the surface, boundary impedance condition, green), and deposit (2D mesh, TCS surfacic model, red).

Table II: Material properties (conductivity σ (S/m) and relative permeability μ_r) of the Tube, Tube Support Plate and deposit.

	σ (S/m)	μ_r
Tube (Inconel 600)	$0.97\,10^6$	1.01
Tube Support Plate (steel)	$1.75\,10^6$	70
Deposit ($50\mu m$)	61	1.64

IV. Numerical Example

We consider a tube of a steam generator, with a thin conductive layer on its outside surface, inserted in a quadrifoiled tube support plate. The mesh of the problem is shown on Figure 2, and material properties in Table II. Note that we use an impedance boundary condition to model the tube support plate. The probe, made of two coaxial coils, works at $100kHz$. We show on Fig. 3 the numerical results obtained.

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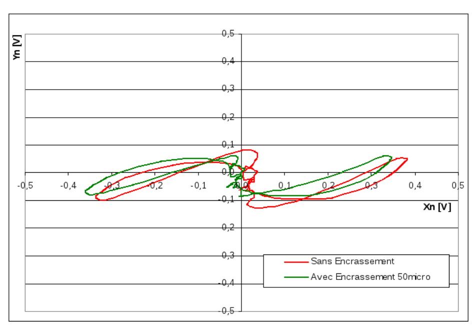

Figure 3: Influence of the conductive deposit on the external surface of the tube on the (Lissajoux) signal received by the probe: without/with deposit (red/green)