Parallel Implementation for Mortar Finite Element Method in Electrostatic Problems

Liao Caibo, Ruan Jiangjun,*Member,IEEE*, Du Zhiye, Wangdong, Liu Shoubao, Liu Chao
School of Electrical Engineering, Wuhan University
No.8, South Road of Eastern Lake, 430072, China
duzhiye@126.com

***Abstract*—The block symmetric coefficient matrix obtained by non-overlapping mortar finite element method (NO-MFEM) is suitable for parallel computing. The main procedure and data storage strategy in parallel NO-MFEM are discussed in the paper. Then, the parallel implementation of NO-MFEM to calculate the potential distribution of DC voltage divider is discussed in the paper. The parallel computing result is in good agreement with NO-MFEM in serial mode, and has significant superiorities in modelling and solving of finite element analysis compared with serial one.**

***Index Terms*—Parallel Algorithms, Domain Decomposition, Mortar Finite Element Method, DC Voltage Divider.**

I. INTRODUCTION

The mortar element method was introduced in for domain decompositions in order to couple different discretization in different sub-domains[1]-[3]. The main feature of mortar element method is to approximate the original continuous function space by discretized mortar space, and the continuity of degree of freedoms across the non-conforming interface is ensured by the surface integration in a weak sense[4].

Parallel computing has become a new solution choice for numerical problem in many research fields, and the block symmetric coefficient matrices obtained by non-overlapping mortar finite element method make it possible for parallel implementation of NO-MFEM[5],[6]. The main procedures of parallel NO-MFEM are introduced in the paper, modeling and discretization of each sub-domain are completed in different processors, and the mortar condition of the interface ensures message exchange between processors in parallel computing. Thus the partition of sub-domains and the message exchange between different processor in parallel implementation of NO-MFEM is discussed.

Making use of parallel NO-MFEM discussed in this paper, the potential distribution of DC voltage divider is calculated. By parallel NO-MFEM, the whole model of DC voltage divider is divided into two sub-domains, and only the sub-domain includes the grading rings needs to be changed and re-meshed in the optimization of grading rings of the DC voltage divider. The parallel result and computing performance is displayed in the end of the paper.

II. PARALLEL MORTAR FINITE ELEMENT METHOD

A. Mortar Finite Element Method

A model with two sub-domains and one internal interface by non-overlapping mortar finite element is discussed in the paper, for the description of the model see Fig.1. Ω_1 is non-mortar side, Ω_2 is mortar side, and γ is the interface of two sub-domains.

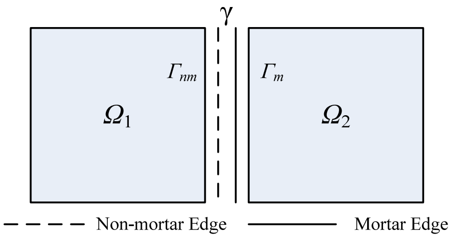


Fig.1. NO-MFEM model with two sub-domains

According to Galerkin finite element method, the equations in sub-domain Ω_1 and Ω_2 can be described as (1).

$$\begin{bmatrix} \mathbf{K}^1_{\Gamma\Gamma} & \mathbf{K}^1_{\Gamma i} & & \\ \mathbf{K}^1_{i\Gamma} & \mathbf{K}^1_{ii} & & \\ & & \mathbf{K}^2_{\Gamma\Gamma} & \mathbf{K}^2_{\Gamma i} \\ & & \mathbf{K}^2_{i\Gamma} & \mathbf{K}^2_{ii} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{1\Gamma} \\ \mathbf{u}_{1i} \\ \mathbf{u}_{2\Gamma} \\ \mathbf{u}_{2i} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{1\Gamma} \\ \mathbf{F}_{1i} \\ \mathbf{F}_{2\Gamma} \\ \mathbf{F}_{2i} \end{bmatrix} \tag{1}$$

Where, $u_{1\Gamma}$ and $u_{2\Gamma}$ correspond to degree of freedom of nodes on interface Γ_{nm} and Γ_m, u_{1i} and u_{2i} to the rest nodes in sub-domain Ω_1 and Ω_2.

The mortar condition denoted as (2) ensure the continuity of degree of freedoms on interface γ between two sub-domains.

$$\mathbf{u}_{1\Gamma} = \mathbf{Q}\mathbf{u}_{2\Gamma} \tag{2}$$

With $u_{1\Gamma}$ in (1) replaced by $u_{2\Gamma}$ according to (2), the full matrix of NO-MFEM based on two sub-domains and one interface is

$$\begin{bmatrix} \mathbf{K}^1_{ii} & \mathbf{K}^1_{i\Gamma}\mathbf{Q} & \mathbf{0} \\ \mathbf{Q}^T\mathbf{K}^1_{\Gamma i} & \mathbf{Q}^T\mathbf{K}^1_{\Gamma\Gamma}\mathbf{Q} + \mathbf{K}^2_{\Gamma\Gamma} & \mathbf{K}^2_{\Gamma i} \\ \mathbf{0} & \mathbf{K}^2_{i\Gamma} & \mathbf{K}^2_{ii} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{1i} \\ \mathbf{u}_{2\Gamma} \\ \mathbf{u}_{2i} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{1i} \\ \mathbf{Q}^T\mathbf{F}_{1\Gamma} + \mathbf{F}_{2\Gamma} \\ \mathbf{F}_{2i} \end{bmatrix}. \tag{3}$$

Where, the coefficient matrix in (3) is positive definite and block symmetric suitable for parallel computing.

B. Parallel Computing Process

Sub-domain partition, modeling and parallel computing are the principle processes of parallel mortar finite element method, as denoted in Fig.2. Sub-domain partition is artificial determined by researchers, while the procedures displayed in dotted box are completed by different processors, and has significant superiorities compared to serial NO-MFEM.

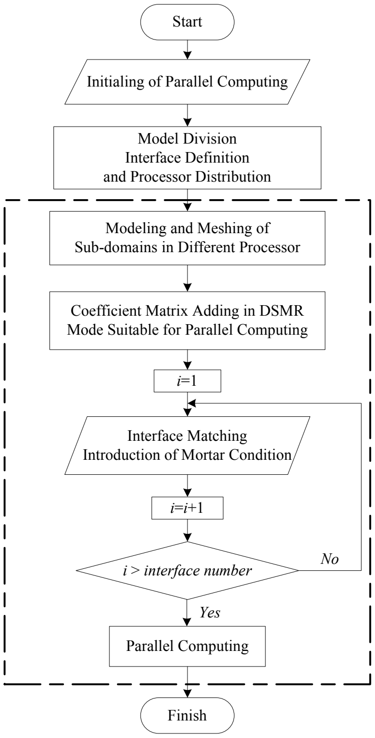


Fig.2. Procedures of Parallel NO-MFEM

C. Node Sets Division

For implementation of parallel computing, the definition of node sets in non-mortar side by parallel NO-MFEM is shown in Fig.3. While the definition of node sets for Processor P' is opposite, with the master nodes on interface in mortar side.

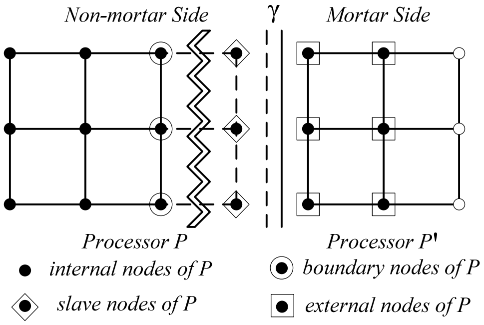


Fig.3. Definition of node sets for Processor P

D. Data Storage Strategy

In order to ensure the continuity of degree of freedoms on interface, the message of mortar condition should be shared between different processors, including

1) node number message of different sub-domain;
2) boundary nodes message of different processors;
3) master and slave nodes message on interface.

III. IMPLEMENTATION OF PARALLEL NO-MFEM

A. DC Voltage Divider

For a detail illustration of DC voltage model see Fig.4, and a 2D axis symmetric model is built. Sub-domain partition is determined by author, and the parallel computing programs of NO-MFEM is self-compiled.

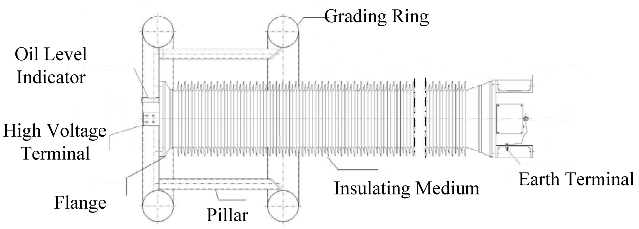
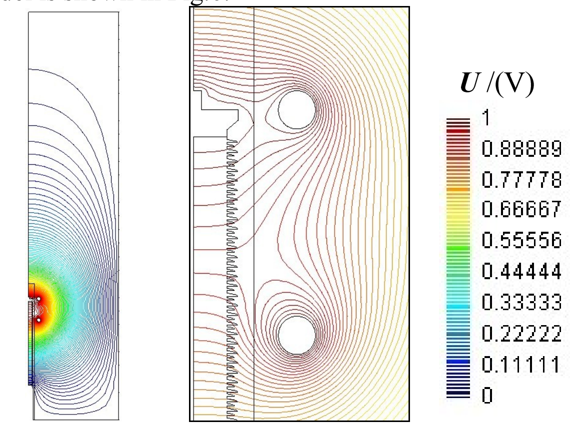


Fig.4. Structure of DC voltage divider

B. Parallel Computing Results

By parallel computation, the potential distribution result of the model is shown in Fig.6.



(a) Full model (b) Partial result nearby interface

Fig.6. Potential distribution contour

C. Contrast Analysis

Distributed parallel system consists of two NP110 Servers connected by a gigabit ethernet, and each server is made up of Pentium 3.0GHZ processor, 2G RAM and other I/O devices. Parameters of parallel NO-MFEM is shown in Table I, the parallel solution time is 55% compared to serial computing.

TABLE I Parameters of Parallel No-MFEM

Performance Index		Parameters
Parallel Solver		CGS
Iterative Steps		107
Solution Time	Parallel	4.29s
	Serial	7.80s

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