MPI Parallel Scheme of 3D Time Domain Boundary Element Method with CRS Matrix Compression

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***Abstract*—A time domain boundary element method (TDBEM) has some original features in time domain microwave simulation, which are not found in a finite difference time domain (FDTD) method., owing to surface meshing. However, it is known that the three-dimensional TDBEM requires extremely large memory compared with the FDTD method. In this paper, efficient memory saving method by a CRS matrix compression which is suitable for a MPI parallel processing of the TDBEM is presented.**

***Index Terms*—Microwave propagation, Parallel processing, Moment methods, Numerical simulation**

I. INTRODUCTION

In microwave simulations, a time domain boundary element method (TDBEM) [1]-[3] provides us some attractive possibilities such as open boundary problems, treatment of slightly curved surface shapes, etc. compared with a finite difference time domain (FDTD) method. On the other hand, most of these applications can be treated by three-dimensional formulation of the TDBEM, and it is known that the three-dimensional TDBEM requires extremely large memory. For example, the required memory exceeds 320GB even if the number of surface meshes is 10,000. This situation has been preventing the TDBEM from being used in practical applications. In accordance with recent remarkable popularization of parallel calculation system by PCs with over 32GB memory, it seems that three-dimensional simulation by the TDBEM is also available for the practical use. To support such the microwave simulation by 3D TDBEM, an efficient memory saving method using a compressed row storage (CRS) method [4] and a MPI parallel processing adjusted to the CRS scheme are presented in this paper.

II. TIME DOMAIN MFIE AND TDBEM MATRIX EQUATION

A time domain electromagnetic fields, $\mathbf{B}(t,\mathbf{x})$, $\mathbf{E}(t,\mathbf{x})$, in a domain V can be expressed using electromagnetic fields on the domain surface $S=\partial V$ in the following surface integral equation forms, if we assume that the boundary S is a perfectly electric conductor (PEC) throughout [3];

$$\begin{aligned}\mathbf{B}(t,\mathbf{x}) &= \mathbf{B}_{ext}(t,\mathbf{x}) \\ &\quad + \frac{1}{4\pi}\int_S \left\{ \left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right) \times \left(\mathbf{n}'\times\mathbf{B}(t',\mathbf{x}')\right) \right\} dS'.\end{aligned} \tag{1}$$

$$\begin{aligned}\mathbf{E}(t,\mathbf{x}) &= \mathbf{E}_{ext}(t,\mathbf{x}) + \frac{1}{4\pi}\int_S \left\{ \frac{\mathbf{n}'\times\dot{\mathbf{B}}(t',\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} \right\} dS' \\ &\quad + \frac{1}{4\pi}\int_S \left\{ -\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right) \left(\mathbf{E}(t',\mathbf{x}')\cdot\mathbf{n}'\right) \right\} dS'\end{aligned} \tag{2}$$

where $\mathbf{B}_{\mathrm{ext}}(t,\mathbf{x})$ and $\mathbf{E}_{\mathrm{ext}}(t,\mathbf{x})$ are externally applied magnetic and electric fields, respectively, the retarded time t ' is defined by $t'=t-|\mathbf{x}-\mathbf{x}'|/c$, c is the velocity of the light and $\mathbf{n}$ is a unit normal vector on the surface. Tangential components of the magnetic field $\mathbf{n}\times\mathbf{B}$ and a normal component of the electric field $\mathbf{n}\cdot\mathbf{E}$ on the domain surface correspond to a surface current density $\mathbf{K}$ and a surface charge density σ, respectively. We can obtain a matrix equation of the TDBEM (Fig.1) to discretize Eqs.(1) and (2) in time and surface, and find the time domain behavior of the surface current and charge densities to solve the matrix equation. In the TDBEM matrix equation of Fig.1, unknown vectors at different time steps are independent each other owing to the retarded time property of Eqs.(1) and (2). Therefore we need to store all coefficient matrices G_l of Fig.1 during the time domain calculation. Then, it is known that the TDBEM matrices G_i are sparse and the required memory can be reduced employing a compressed storage. Fig.2 shows relation between causality lines and the TDBEM matrices. It is found that the coefficients in the integrand of (1) are distributed in the matrices of Fig.1 depending on a distance between an observation and referred points | $\mathbf{x}$ - $\mathbf{x}'$| and a unit discretized time step Δt, owing to the retarded property. If we assume that the number of surface meshes is $N = n^2$ (n is one dimension size of the surface mesh) and the number of the matrices in Fig.1 is L (roughly peaking it is same order as n, since $c\Delta t$ is taken to be the same size as a typical mesh size Δl), the total size of the required memory M

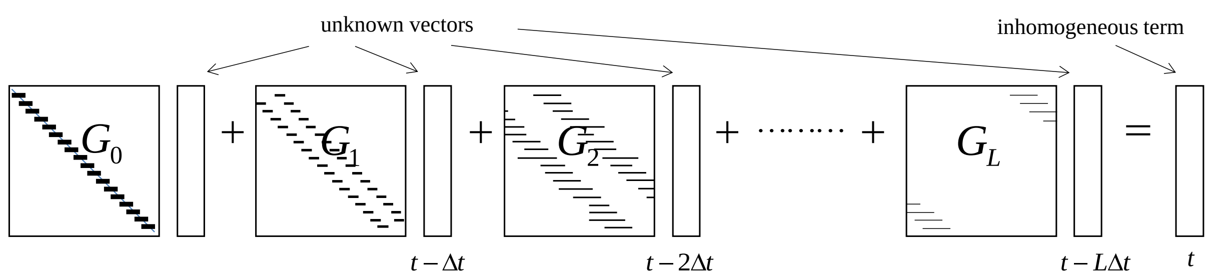

Fig. 1. Configuration of matrix equation of TDBEM

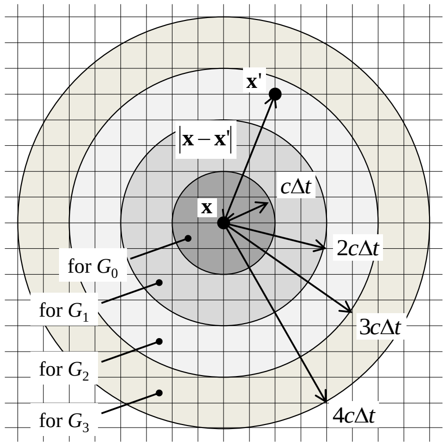

Fig. 2. Relation between causality line and the TDBEM matrices

is estimated as $M = n^5 \times 2^2 \times 8$ byte (it is considered that there are two unknowns at the each mesh), if we do not employ any matrix compression schemes. For example, M exceeds to 320GB even for relatively small size n = 100.

III. Compressed Row Storage in TDBEM

It is easily supposed from Fig.2 that the length of non-zero row components in each column of the matrices in Fig.1 has various sizes. In addition, the length of the non-zero row components has various sizes even within the same matrix G_i, depending on a shape of a numerical model. Accordingly memory saving would be very inefficient if we employ a matrix compression method using fixed size matrix array. (see Fig.3(a)) We here employ so-called compressed row storage (Fig.3(b)) [4],[5] to achieve efficient memory saving in 3D TDBEM. Then we need to consider a modification of the CRS method, which is dedicated to the TDBEM, owing to the multi-matrices structure of the TDBEM matrix equation (Fig.1). Fig.4 shows the modified CRS method for the TDBEM, that is, the non-zero matrix components are stored in same order of the matrix equation construction from the integral equations (1) and (2). This means that the process of the matrix construction can be parallelized, which is also suitable for the MPI parallel processing of the TDBEM.

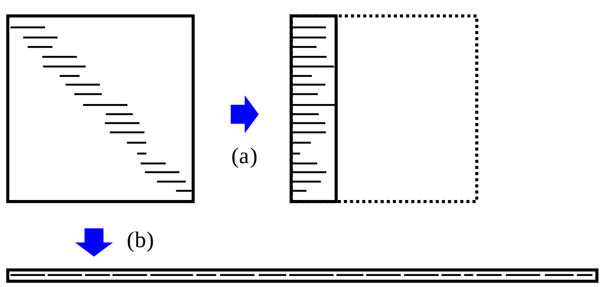

Fig. 3. Configuration of matrix compression methods
(a) fixed size array (b) CRS

In Fig.5, an example of numerical model of a part of a particle accelerator, a simplified model of a collimator section, is indicated. Owing to complicated 3D structure of this model, the memory saving by the matrix compression will not work well if we employ the fixed size array (Fig.2(a)). The required memory size of this model would be about 7TB if we would not employed any matrix compression. Then, the required memory is reduced to about 900GB by the fixed size array method, on the other hand, 200GB by the proposed method.

IV. Summary

A modified compressed row storage method for the TDBEM has been proposed to achieve effective memory saving in this paper. To take into account that the required memory size of 3D TDBEM easily exceeds several hundred GB even for small numerical model, the parallel processing is essential for 3D TDBEM. It is shown that the proposed modified CRS method provide us very efficient memory saving, which is suitable for the MPI parallel processing.

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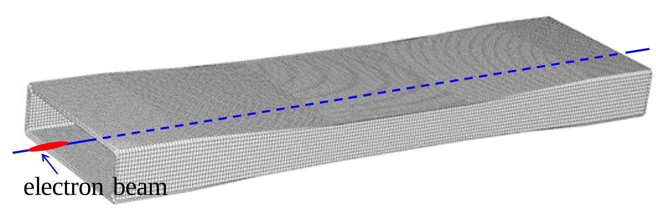

Fig. 5. Numerical model of a part of particle accelerator

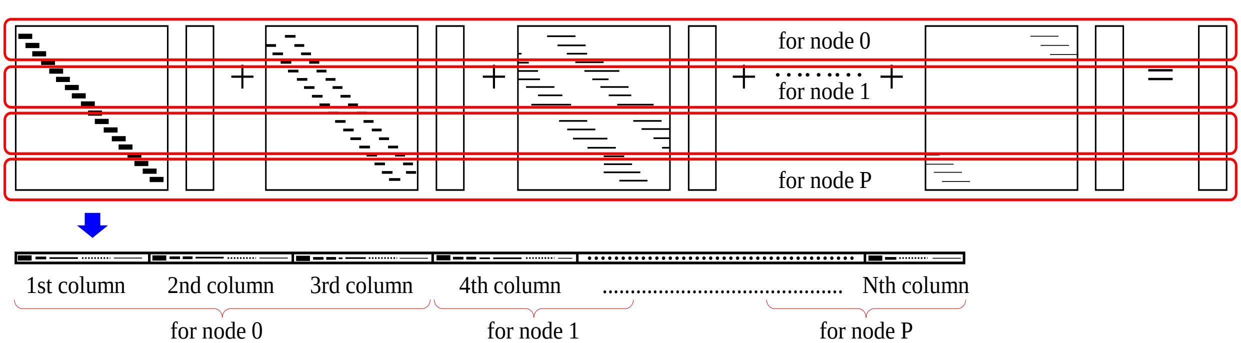

Fig. 4. Modified CRS for MPI parallel processing of TDBEM