Multi-objective Optimization Approach to Reliability-Based Optimal Design of Electromagnetic Problems

Ziyan Ren, Dianhai Zhang, and Chang-Seop Koh, *Senior Member*, *IEEE*
College of Electrical and Computer Engineering, Chungbuk National University, Chungbuk, 361-763, Korea
E-mail: renziyan@chungbuk.ac.kr

***Abstract*—For the optimal design of electromagnetic device with uncertainty, this paper proposes a new multi-objective reliability-based design optimization (RBDO) algorithm by optimizing the performance and maximizing the minimum reliability. Furthermore, the second-order sensitivity assisted Monte Carlo simulation (SSA-MCS) method is suggested for reliability analysis, where the second-order design sensitivity is implemented by a hybrid direct differentiation-adjoint variable method. The proposed multi-objective RBDO algorithm is investigated through application to the optimal design of a superconducting magnetic energy storage system.**

***Index Terms*— Multi-objective optimization, reliability, sensitivity-assisted Monte Carlo simulation, sensitivity analysis.**

I. INTRODUCTION

In the electrical engineering, considerable robust design algorithms attempt to minimize performance variations [1]. The algorithm, which can guarantee the specific feasibility level in the probabilistic sense against uncertainty, however, has been scarcely presented. It deserves to be mentioned that the reliability-based design optimization (RBDO) from mechanical engineering, tries to improve the reliability of a design under uncertainty at the required level [2].

Recently, also in electrical engineering, attention is paid to the reliability assessment and the RBDO [3]. The preliminary investigation only mentions the reliability index approach, of which the computational effort is very expensive for problems involving performance analysis by the finite element method (FEM). Therefore, in our previous research [4], we proposed a first-order sensitivity-assisted Monte Carlo simulation (FSA-MCS) to efficiently evaluate reliability. However, it looks insufficient when researched problems involve bigger uncertainties or performance functions are strongly nonlinear.

In this paper, for intensive study on the RBDO, the reliability metric is incorporated into a multi-objective RBDO problem. To improve the accuracy of the FSA-MCS method, the second-order sensitivity is studied by the hybrid direct differentiation-adjoint variable method.

II. RELIABILITY CALCULATION AND RBDO ALGORITHMS

A. Reliability Calculation

Firstly, in the uncertainty set $U(\mathbf{x})$ of a nominal design $\mathbf{x}$ [4], the performance constraint ($g(\mathbf{x})\leq 0$) at the test design $\boldsymbol{\xi}$ is approximated by using Taylor expansion as:

$$g(\boldsymbol{\xi}) \cong g(\mathbf{x}) + \nabla g(\mathbf{x})(\boldsymbol{\xi}-\mathbf{x}) + (\boldsymbol{\xi}-\mathbf{x})^T H(\mathbf{x})(\boldsymbol{\xi}-\mathbf{x})/2 \qquad (1)$$

where $\nabla g(\mathbf{x})$ and $H(\mathbf{x})$ are the gradient vector and the Hessian matrix, respectively. The reliability is approximated as [4]:

$$R\left(g(\mathbf{x})\leq 0\right) = \left(1/N\right)\cdot \sum_{i=1}^{N} I\left[g(\xi_i)\right] \qquad (2)$$

where N is the number of random test designs and $I[\ ]$ is the indicator function. The numerical implementation of the SSA-MCS is shown in Fig. 1.

B. Second-Order Sensitivity Analysis Based on FEM

In the FEM, the system matrix can be written as:

$$[K][A] = \{Q\} \qquad (3)$$

where $[A]$ is the magnetic vector potential; $[K]$ and $[Q]$ are the stiffness matrix and the forcing vector, respectively. Based on (3), the second-order sensitivity of function $g(\mathbf{x})$ with respect to design parameters p_i and p_j is derived as follows:

$$\begin{aligned}\frac{d^2 g}{dp_i dp_j} &= \frac{\partial^2 g}{\partial p_i \partial p_j} + \frac{\partial^2 g}{\partial p_i \partial [A]^T}\frac{d[A]}{dp_j} + \frac{\partial^2 g}{\partial [A]^T \partial p_j}\frac{d[A]}{dp_i} \\ &\quad + \frac{d[A]^T}{dp_j}\frac{\partial^2 g}{\partial [A]^2}\frac{d[A]}{dp_i} - [\lambda]^T \left\{ \frac{\partial^2 [K]}{\partial p_i \partial p_j}[\tilde{A}] \right. \\ &\quad \left. - \frac{\partial^2 \{Q\}}{\partial p_i \partial p_j} + \frac{\partial [K]}{\partial p_i}\frac{d[A]}{dp_j} + \frac{\partial [K]}{\partial p_j}\frac{d[A]}{dp_i} \right\}\end{aligned} \qquad (4)$$

where $[\tilde{A}]$ is the converged solution of (3), $[p]$ is related with nodal mesh, and $[\lambda]$ is adjoint variable defined as follows:

$$[K][\lambda] = \partial f / \partial [A]. \qquad (5)$$

From (3), the derivative $d[A]/dp_i$ needed in (4) is obtained by solving equation:

$$[K]\frac{d[A]}{dp_i} = \frac{\partial \{Q\}}{\partial p_i} - \frac{\partial [K]}{\partial p_i}[\tilde{A}]. \qquad (6)$$

In addition, other terms related with the constraint function are problem dependent and can be evaluated analytically. The above algorithm is the hybrid direct differentiation-adjoint variable method [5] due to usages of (5) and (6). Its computational cost is linearly proportional to the number of design parameters (t) and needs (t+2) times of FEM calls [5].

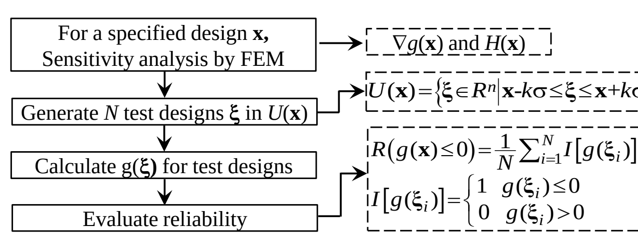

Fig. 1. Flowchart of the second-order sensitivity-assisted MCS method.

C. Multiobjective Reliability-Based Design Optimization

The conventional RBDO problem is formulated as:

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{subject to} & R\left(g_i(\mathbf{x}) \le 0\right) \ge R_i^t, \quad i = 1, \cdots, m. \end{array} \tag{7}$$

where R^t is the predefined target reliability and m is the number of constraint function. Solving (7), one reliable solution can be obtained. In the real problem, however, it is more significant to learn how the reliable solutions change with different reliability values. Therefore, a multi-objective RBDO (MO-RBDO) problem is proposed as:

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{maximize} & R_{\min} = \min\left\{R\left(g_i(\mathbf{x}) \le 0\right)\right\}, i = 1, \cdots, m \end{array} \tag{8}$$

where R_{min} is the minimum reliability among all constraints.

Obviously, algorithm (8) can supply a Pareto-optimal set by making a tradeoff between the performance and reliability.

III. Numerical Result Of Superconducting Magnetic Energy Storage System (SMES)

The SMES is selected as the 22nd problem for testing of electromagnetic analysis method (TEAM 22) [6], In the 3-parameter TEAM 22 [2], $\mathbf{x}=[R_2, H_2/2, D_2]^T$ m, the objective function and constraints are:

$$f(\mathbf{x}) = \frac{B_s^2}{B_n^2} + \frac{\left|E(\mathbf{x}) - E_{ref}\right|}{E_{ref}}, B_s^2 = \frac{1}{22}\sum_{i=1}^{22} B^2(i) \tag{9a}$$

$$g_i(\mathbf{x}) = \left|J_i\right| + 6.4 \cdot \left|B_{m,i}\right| - 54.0 \le 0, \quad i = 1, 2 \tag{9b}$$

where B_n=3 mT and other symbols are explained in [6].

The constraint approximation by design sensitivity is compared in Fig. 2. Table I compares the reliability of different optimal designs. The results from the SSA-MCS are very close to the MCS by giving smaller errors.

The parameters (particles/iterations) in the particle swarm optimization are set as: (30/200) for classical and RBDO problems and (50/300) for MO-RBDO. For reliability analysis, uncertainties in geometric variables are $k\boldsymbol{\sigma}_{\mathbf{x}}$=0.0392 m, $\mathbf{J}$ follow Gaussian distribution with $\boldsymbol{\mu}_{\mathbf{J}}=[16.78,-15.51]^T$ MA/m^2 and $k\boldsymbol{\sigma}_{\mathbf{J}}$=0.45 MA/m^2. During optimization, fixed values are R_1=1.32 m, $H_1/2$=1.07m, and D_1=0.59 m.

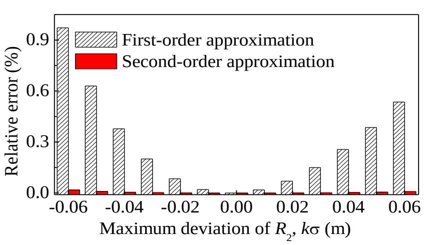

Fig. 2. Approximation of $g_2(\mathbf{x})$ for design $\mathbf{x}=[1.8127,1.4963,0.2458]^T$ m.

TABLE I
Reliability Of Different Optimal Designs

Different design $\mathbf{x}$ [m]			Reliability of $g_2(\mathbf{x})$ [a]		
R_2	$H_2/2$	D_2	FSA-MCS	SSA-MCS	MCS
3.0500	0.2460	0.4000	0.7200	0.7232	0.7231
2.6602	0.5574	0.2218	0.9500	0.9512	0.9521
3.0988	0.2644	0.3903	0.6679	0.6708	0.6716
3.0197	0.3081	0.3496	0.5158	0.5215	0.5210

[a] Reliability of $g_1(\mathbf{x})$ for all cases is 1.0, $k\boldsymbol{\sigma}_{\mathbf{x}}=[0.03, 0.0196, 0.0196]^T$ m and test designs in all MCSs are set 10,000.

Table II compares optimization results of the RBDO and the classical optimum, where the number of test points in the SSA-MCS is 1,000,000. It is found that the classical optimum has very low reliability and has higher possibility, in this case 46.33%, to violate the constraint $g_1(\mathbf{x})\le 0$. As the target reliability increases, the optimal design of the RBDO gives a little worse objective value such as R^t=0.6 and 0.7; however, it locates further inside the feasible region by giving bigger margins to both constraints.

Fig. 3 shows the optimization result of the proposed MO-RBDO method together with optimum in Table II. It is found that the Pareto-front includes optimums obtained by the RBDO, and design A (one of the extreme solutions) is very similar to the classical optimal design. The Pareto front also provides important information to make a balance between the objective function and minimum reliability according to different requirements. If the constraints are extremely critical, design C with bigger reliability may be selected although it has very poor performance. Design B may be considered as a better solution in the general-purpose optimization since it makes a good trade-off between performance and reliability. Fig. 4 shows constraint values of each Pareto optimum, which reveals that a design with smaller margins for constraints will result in a lower reliability in Fig. 3 such as design A.

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TABLE II
Optimal Results Of Reliability-Based Design Optimization

R^t	R_2	$H_2/2$	D_2	$f(\mathbf{x})$	B_s^2 [T^2]	$g_1(\mathbf{x})$ [a]	$R(g_1(\mathbf{x})\le 0)$
Classical	1.8127	1.4963	0.2458	6.226E-5	7.522E-11	-0.1191	0.5367
0.60	1.8229	1.4061	0.2592	3.076E-4	2.748E-9	-0.3534	0.6375
0.70	1.8121	1.4885	0.2519	1.358E-2	6.511E-8	-0.5918	0.7209

[a] All designs have enough margins for constraint $g_2(\mathbf{x})\le 0$.

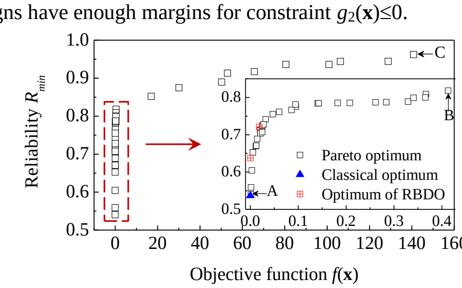

Fig. 3. Optimal results of the proposed MO-RBDO and the RBDO.

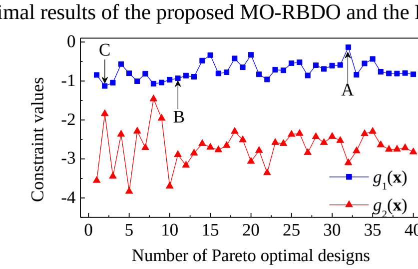

Fig. 4. Constraint values of each Pareto-optimal design.