Comparative Study of Reliability Evaluation Methods for Reliability-based Design Optimization of Electromagnetic Devices under Uncertainty

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***Abstract*—In order to find an efficient and accurate reliability calculation method for the electromagnetic problem, this paper presents a comparative study among different algorithms for reliability analysis. For the accuracy improvement of the sensitivity-assisted Monte Carlo simulation method, a hybrid direct differentiation-adjoint variable method is applied to calculate the second-order sensitivity in finite element analysis. The performance of each method is validated by application to one superconducting magnetic energy storage (SMES) system.**

***Index Terms*—Constraint feasibility, design sensitivity analysis, reliability calculation.**

I. Introduction

Recently, a reliability-based design optimization (RBDO) has been applied to the optimal design of electromagnetic devices to maintain the design feasibility even with uncertain design variables [1],[2].

For the reliability analysis, there are mainly two kinds of methods generally applied in the mechanical engineering: 1) optimization-based method such as reliability index approach (RIA) [1] and performance measure approach (PMA) [3]; 2) sampling-based method such as Monte Carlo simulation (MCS) [3]. Both the RIA and the PMA need an independent optimization loop, while the MCS needs as much samples as possible (one million). The electromagnetic problem, however, normally involves a performance analysis by numerical method such as the finite element method (FEM). To widely apply the RBDO in the electrical engineering, it is essential to investigate characteristics of different reliability algorithms and find one efficient reliability assessment algorithm.

In the previous research [2], we proposed a sensitivity-assisted Monte Carlo simulation (SA-MCS) to fast evaluate reliability. Due to the fist-order approximation, however, it looks insufficient when the researched problems involve bigger uncertainties or the performance constraint functions are strongly nonlinear.

This paper makes a comparative investigation of reliability analysis. Furthermore, for improvement of the SA-MCS, the second-order sensitivity-assisted MCS method is proposed, where the second-order design sensitivity is implemented by the hybrid direct differentiation-adjoint variable method.

II. Reliability Analysis

Hereinafter, all variables are treated as uncertain ones and independently follow Gaussian distribution with means ($\boldsymbol{\mu}$), and standard deviations ($\boldsymbol{\sigma}$), $\mathbf{x}\sim N(\boldsymbol{\mu}, \boldsymbol{\sigma})$. The domain decided by constraint $g(\mathbf{x})\le 0$ represents the feasible region. The direct definition of reliability involves multi-dimensional integral of probabilistic distribution, which is very difficult. As a result, many equivalent approximations have come into being.

A. Reliability Index Approach

The reliability analysis of RIA is implemented as follows:

Step 1: Transfer the uncertain variables ($\mathbf{x}$) into the standard normalized ones ($\mathbf{u}$) using:

$$\mathbf{u}=(\mathbf{x}-\boldsymbol{\mu})/\boldsymbol{\sigma}. \tag{1}$$

Step 2: Find the most probable point of failure (MPPF) on the constraint curve by solving one optimization problem [1]:

$$\begin{aligned}&\text{Minimize} \quad \|\mathbf{u}\| \\ &\text{subject to} \quad g(\mathbf{u})=0.\end{aligned} \tag{2}$$

Step 3: Calculate reliability index β and reliability R:

$$R=1-\Phi(-\beta), \quad \text{and} \quad \beta=\|\mathbf{u}^*\| \tag{3}$$

where $\mathbf{u}^*$ is the optimal solution of (2) and Φ() is the standard normal density function.

B. Performance Measure Approach

Compared with the RIA, the MPPF of the PMA under a specified reliability index β_t is obtained by solving problem:

$$\begin{aligned}&\text{Maximize} \quad g(\mathbf{u}) \\ &\text{subject to} \quad \|\mathbf{u}\|=\beta_t.\end{aligned} \tag{4}$$

The PMA is faster than the RIA, however, it only determines whether a solution is reliable or not against constraint satisfaction with respect to a specified reliability index.

C. Sensitivity-Assisted Monte Carlo Simulation

For a specified design $\mathbf{x}$, the reliability with respect to the performance constraint $g(\mathbf{x})\le 0$, is evaluated as follows:

Step 1: Execute performance and sensitivity analysis by the FEM for uncertain design variables.

Step 2: In the uncertainty set $U(\mathbf{x})$ defined in [2], generate N test designs ($\boldsymbol{\xi}$) following the probability distribution.

Step 3: Calculate constraint of each test design by using:

$$g(\boldsymbol{\xi})=g(\mathbf{x})+\nabla g(\mathbf{x})\cdot(\boldsymbol{\xi}-\mathbf{x})+\cdots \tag{5}$$

where $\nabla g(\mathbf{x})$ is the gradient vector.

Step 4: Evaluate reliability $R(g(\mathbf{x})\le 0)$ by the MCS as:

$$R\big(g(\mathbf{x})\le 0\big)=\sum_{i=1}^{N} I\big[g(\boldsymbol{\xi}_i)\big]\big/N \tag{6a}$$

$$I\big[g(\boldsymbol{\xi}_i)\big]=1 \quad if \ g(\boldsymbol{\xi}_i)\le 0. \tag{6b}$$

If only the first-order derivative is included in (5), the above method is called the first-order sensitivity-assisted MCS (FSA-

MCS); To obtain higher accuracy, we develop the second-order design sensitivity by the FEM, and suggest the second-order sensitivity-assisted MCS (SSA-MCS) approach.

III. Second-order Sensitivity Analysis by FEM

The second-order sensitivity of constraint $g(\mathbf{x})\leq 0$ with respect to design parameters p_i and p_j is derived as follows:

$$\begin{aligned}\frac{d^2 g}{dp_i dp_j} &= \frac{\partial^2 g}{\partial p_i \partial p_j} + \frac{\partial^2 g}{\partial p_i \partial [A]^T}\frac{d[A]}{dp_j} + \frac{\partial^2 g}{\partial [A]^T \partial p_j}\frac{d[A]}{dp_i} \\ &+ \frac{d[A]^T}{dp_j}\frac{\partial^2 g}{\partial [A]^2}\frac{d[A]}{dp_i} - [\lambda]^T \left\{ \frac{\partial^2 [K]}{\partial p_i \partial p_j}[\tilde{A}] \right. \\ &\left. - \frac{\partial^2 \{Q\}}{\partial p_i \partial p_j} + \frac{\partial [K]}{\partial p_i}\frac{d[A]}{dp_j} + \frac{\partial [K]}{\partial p_j}\frac{d[A]}{dp_i} \right\}\end{aligned} \quad (7)$$

where $[A]$ is the magnetic vector potential and $[\tilde{A}]$ is the converged solution of system equation $[K][A]=\{Q\}$; $[K]$ and $\{Q\}$ are the stiffness matrix and the forcing vector, respectively; $[p]$ is related with nodal mesh. The derivative $d[A]/dp_i$ in (7) is obtained by applying the direct differentiation method to $[K][A]=\{Q\}$ as follows:

$$[K]\frac{d[A]}{dp_i} = \frac{\partial\{Q\}}{\partial p_i} - \frac{\partial [K]}{\partial p_i}[\tilde{A}] \quad (8)$$

In (7), the adjoint variable $[\lambda]$ is obtained by solving:

$$[K][\lambda] = \partial g / \partial [A]. \quad (9)$$

In addition, constraint function related terms are problem dependent and can be evaluated analytically. The above algorithm is the hybrid direct differentiation-adjoint variable method due to usages of (8) and (9). Its computational complexity is linearly proportional to the number of design parameters (t) and needs (t+2) FEM calls [4].

IV. Numerical Calculation Results

A. Analytic Example

For performance comparison, a constraint function with strong nonlinearity is selected as follows:

$$g(\mathbf{x}) = -1 + (s-6)^2 + (s-6)^3 - 0.6(s-6)^4 + t \geq 0 \quad (10)$$

where $0\leq x_1, x_2 \leq 10$, $s=ax_1+bx_2$, and $t=bx_1-ax_2$ (a=0.9063, b=0.4226). In the strongly nonlinear area, three different designs are selected as marked in Fig. 1. The reliability analyses by different methods are compared in Table I. Taking the reliability of the conventional MCS method as a reference R_0, and the relative error (δ_R) of reliability R from other methods is defined as $\delta_R=|R-R_0|/R_0\times 100\%$. From Table I, it is obvious that due to the linear approximation, the FSA-MCS and the RIA cannot give the accurate reliability even under a small uncertainty such as $\boldsymbol{\sigma}$=0.2, however, the SSA-MCS can still give a higher accuracy with the maximum relative error of 3.87% when uncertainty is increased to $\boldsymbol{\sigma}$=0.3.

Therefore, we can conclude that the second-order sensitivity analysis is very essential for the strong nonlinear constraint function approximation. The SSA-MCS owns wider application space than the RIA and the FSA-MCS. It can be expected to improve the quality of optimal design in the reliability-based design optimization.

B. Superconductivity Magnetic Energy Storage System

For the SMES system [5], quenching condition of superconducting coil is selected as the performance constraint to test reliabilities of published optimal designs [5]-[8]:

$$g_i(\mathbf{x}) = 6.4\cdot\left|B_{m,i}\right| + \left|J_i\right| - 54.0 \leq 0, \quad i=1,2 \quad (11)$$

where the symbols are defined in [5]. The $\mathbf{x} = [R_2, H_2, D_2]^T$ are treated as uncertain variables. The 10,000 maximum test designs and confidence level of 0.95 are applied in the MCS based methods. Table II shows the reliability results with $\boldsymbol{\sigma}$=[15.3, 10, 10]^T (mm). It can be seen that results of the SSA-MCS method match well with the MCS method. The detail characteristics of the PMA will be discussed in the full paper.

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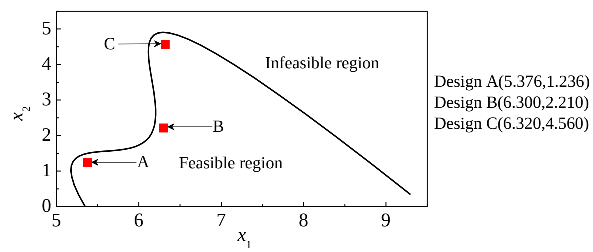

Fig. 1. Analytic example and target designs.

Table I
Comparison of Reliability Calculation [a]

σ	Method	Design A R	Design A δ_R (%)	Design B R	Design B δ_R (%)	Design C R	Design C δ_R (%)
0.1	RIA	0.9612	1.908	0.8817	2.682	0.9757	2.205
	FSA-MCS	0.9914	1.174	0.8863	2.174	0.9999	0.221
	SSA-MCS	0.9830	**0.316**	0.9051	**0.099**	0.9944	**0.331**
	MCS	0.9799	—	0.9060	—	0.9977	—
0.2	RIA	0.8112	5.942	0.7230	3.484	0.8380	6.116
	FSA-MCS	0.8662	13.125	0.7223	3.578	0.9284	17.564
	SSA-MCS	0.7632	**0.327**	0.7471	**0.267**	0.7953	**0.709**
	MCS	0.7657	—	0.7491	—	0.7897	—
0.3	RIA	0.7218	15.952	0.6534	4.905	0.7446	27.282
	FSA-MCS	0.7678	23.341	0.6525	5.036	0.8299	41.863
	SSA-MCS	0.5984	**3.872**	0.6827	**0.640**	0.5994	**2.462**
	MCS	0.6225	—	0.6871	—	0.5850	—

[a] Test designs of the MCSs are 1,000,000 and confidence level is 0.95.

Table II
Result of Reliability Calculation

Ref	Optimal design **x** [m]			Reliability of $g_2(\mathbf{x})$ [a]			
	R_2	H_2	D_2	RIA	FSA-MCS	SSA-MCS	MCS
[5]	3.08	0.478	0.394	0.9899	0.9805	0.9807	0.9807
[6]	3.05	0.492	0.400	0.7249	0.7200	0.7232	0.7231
[7]	3.0988	0.5287	0.3903	0.6770	0.6679	0.6708	0.6716
[8]	3.0197	0.6162	0.3496	0.5169	0.5158	0.5215	0.5210

[a] Reliability of $g_1(\mathbf{x})$ for all cases is 1.0.