A Novel Approach of Sensitivity Analysis in Finite Element Method and Its Application

Lin Yang[1], S. L. Ho [1], Weinong Fu[1], Lei Liu[2]
[1] Department of Electrical Engineering, The Hong Kong Polytechnic University, Hong Kong
[2] College of Electrical Engineering, Zhejiang University, Hangzhou, 310027, China

Abstract —This paper presents a novel approach of sensitivity analysis for design optimization of electromagnetic devices based on a parameterized mesh in finite element method (FEM). The proposed method requires no repeat calculation by using the solution of the field equations and the advantage of the parameterized mesh, thus the calculating time is reduced significantly. An optimal design example of an electromagnetic device is reported to positively confirm its feasibilities and advantages.

***Index Terms*—Finite element method, optimal design, parameterized mesh, sensitivity analysis.**

I. Introduction

The importance of sensitivity analysis of electromagnetic devices stems from the need to improve their performance with respect to shape parameters. Based on the sensitivity analysis, the uncertainties between design parameters and objective function can be better estimated.

In this paper, a parameterized mesh generation method is proposed. The initial mesh is created based on Delaunay algorithm [1], and the coordinates of all the nodes are linear functions with respect to geometrical parameters. When new nodes are added, the coordinates can be quickly obtained according to the relationship between them [2]. In addition, the sensitivity and the derivative of objective function versus parameter p can be obtained from previous FEM results by back-substituting the right hand side (RHS) in the field matrix equation. Finally, we evolve the function of the magnetic force, in which we can carry out the sensitivity analysis in time domain.

Compared with existing methods, the proposed approach can reduce the computational burden greatly while the accuracy is guaranteed. Firstly, we only need to solve the finite element problem once to obtain the sensitivity information from the FEM results. Secondly, the mesh has no need to be generated repeatedly when the geometrical parameters changes and the mesh quality is high by using a swapping diagonal technique. Thirdly, the method of sensitivity analysis is a kind of direct approach, which can greatly eliminate the inaccuracy problem.

II. Models and Methods

A. Basic Field Equation

When analyzing magnetic field, the following system equation is obtained in matrix form after using FEM discretization:

$$[S]\{A\}-\{P\}=0 \tag{1}$$

For a nonlinear problem, N-R iteration method is used:

$$[J]\{A\}=\{P\}-[S]\{A\} \tag{2}$$

where $\{A\}$ is the nodal magnetic potential vector; the Jacobian matrix is:

$$[J]=\frac{\partial}{\partial A}([S]\{A\}) \tag{3}$$

or:

$$J=\begin{bmatrix} \frac{\partial f_1}{\partial A_1} & \frac{\partial f_1}{\partial A_2} & \cdots & \frac{\partial f_1}{\partial A_n} \\ \frac{\partial f_2}{\partial A_1} & \frac{\partial f_2}{\partial A_2} & & \vdots \\ \vdots & & \ddots & \\ \frac{\partial f_n}{\partial A_1} & \cdots & & \frac{\partial f_n}{\partial A_n} \end{bmatrix} \tag{4}$$

where:

$$f_m=S_{m1}A_1+S_{m2}A_2+\cdots+S_{mn}A_n,\ m=1,2,\ldots,n. \tag{5}$$

By solving (2), the magnetic state is known and all nodal potentials are obtained.

B. Calculation of Sensitivity

When one geometric or physical parameter p changes, the derivative versus p of the stationary system (1) gives the following set of new equations:

$$\frac{\partial}{\partial p}\{[S]\{A\}-\{P\}\}=\frac{\partial}{\partial A}([S]\{A\})\left\{\frac{\partial A}{\partial p}\right\}+\frac{\partial}{\partial p}([S]\{A\})\Big|_{A=constant}-\frac{\partial}{\partial p}\{P\}=0 \tag{6}$$

or:

$$[J]\left\{\frac{\partial A}{\partial p}\right\}=-\frac{\partial}{\partial p}([S]\{A\})\Big|_{A=constant}+\frac{\partial}{\partial p}\{P\} \tag{7}$$

In (7), the term at the RHS is known. In addition, the coefficient matrices of (2) and (7) are the same. When solving (2), the FEM system matrix has been factorized. Therefore, the $\{\partial A/\partial p\}$ can be obtained by back-substituting RHS easily and quickly.

C. Derivative of an Objective Function versus Design Parameters

During optimal design process, the values of objective function change according to the design parameters. Assuming the magnetic force is the objective function [3][4]:

$$F=F(p,A) \tag{8}$$

The magnetic vector potential $A(A_1, A_2, \ldots, A_N)$ is already determined from (1), so it is an implicit function of $p(p_1, p_2, \ldots, p_M)$ as: $A = A(p)$, The design sensitivity is:

$$\frac{dF}{dp_m} = \left.\frac{\partial F}{\partial p_m}\right|_{A=constant} + \sum_{n=1}^{N}\left(\left.\frac{\partial F}{\partial A_n}\right|_{p_m=constant}\frac{\partial A_n}{\partial p_m}\right) \qquad m=1, \ldots, M \qquad (9)$$

or written in matrix format:

$$\frac{dF}{dp} = \left.\frac{\partial F}{\partial p}\right|_{A=constant} + \left.\frac{\partial F}{\partial A^T}\right|_{p=constant}\frac{\partial A}{\partial p} \qquad (10)$$

where T is the transpose sign.

III. Numerical Example

An electromechanical levitation device is used as a test example. The proposed method is utilized to obtain the derivative of the magnetic force versus the geometric sizes. The device model and the design parameters (p_1, p_2) are shown in Fig. 1.

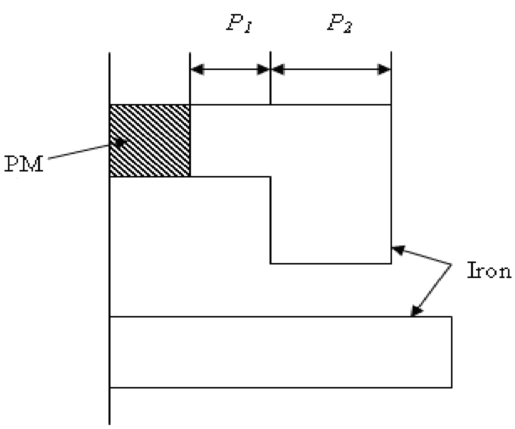

Fig. 1. The model and parameters

During mesh refinement process, when the parameters vary, all coordinates of the refined mesh are changed automatically, which is shown in Fig. 2.

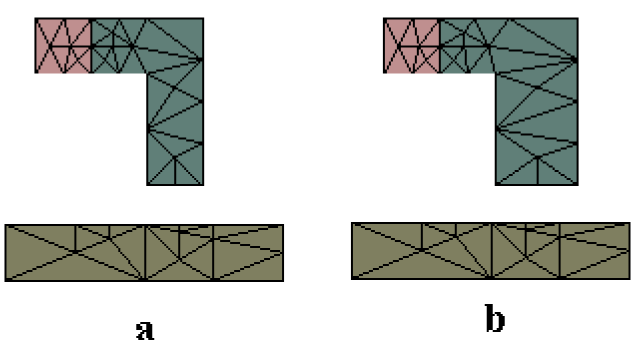

Fig. 2. (a)Initial mesh (b)Refined mesh when parameters change

We use Maxwell stress tensor method to calculate the magnetic force according to the magnetic field on the surface surrounding the objects. In order to overcome the shortage that the force is very sensitive to the quality of the mesh layer surrounding the object, a multi-layer integration method is proposed in this paper.

There are two geometry parameters, namely p_1 and p_2. $0 < p_1 < 10$ mm, $0 < p_2 < 100$ mm. The size of permanent magnet (PM) is fixed to be 2.75 mm. We calculate the derivative versus p_1 100 steps and p_2 1000 steps, respectively, as shown in Fig. 3(a) and (c). The magnetic force with respect to p_1 and p_2 are also shown in Fig. 3 (b) and (d).

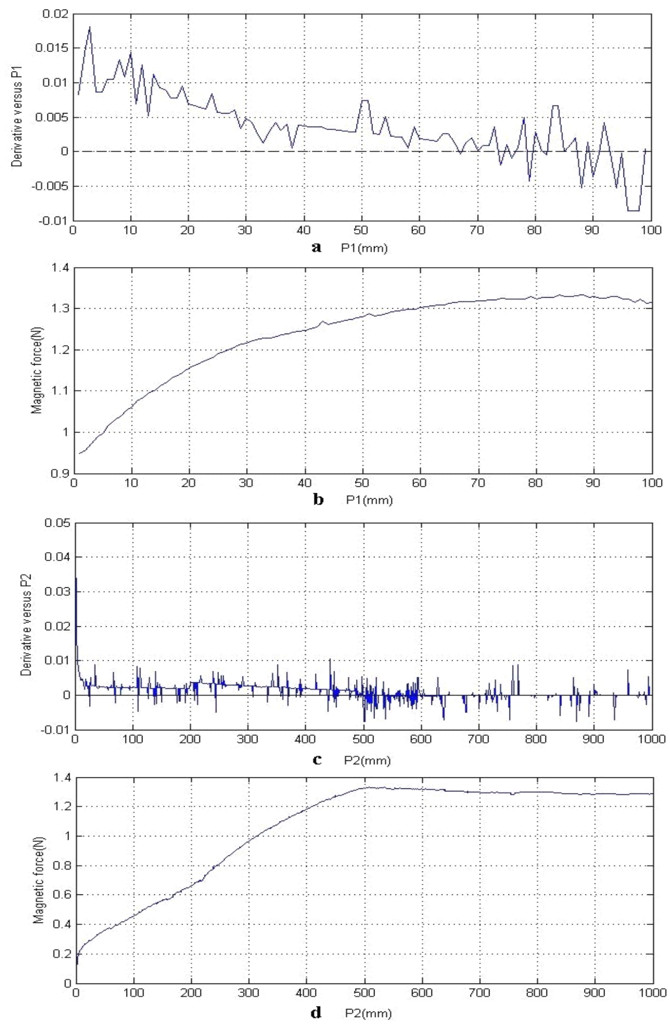

Fig. 3. (a) The derivative versus p_1. (b) The derivative versus p_2. (c) The magnetic force with respect to p_1. (d) The magnetic force with respect to p_2.

References

[1] S. W. Sloan, "A fast algorithm for constructing Delaunay triangulations in the plane," *Advances in Engineering Software,* vol.9, no. 1, pp. 34-55, 1987.

[2] Shuangxia Niu, Yanpu Zhao, S. L. Ho and W. N. Fu, "A parameterized mesh technique for finite element magnetic field computation and its application to optimal designs of electromagnetic device," *IEEE Trans. on Magn.*, vol. 47, no. 10, pp. 2943-2946, 2011.

[3] S. Gitosusastro, J. L. Coulomb and J. C. Sabonnadiere, "Performance derivative calculations and optimization Process," *IEEE Trans. Mag.* vol. 25. NO. 4 JULY 1989, pp. 2834-2839.

[4] J. L. Coulomb, "A methodology for the determination of global electromechanical quantities from a finite element analysis and its application to the evaluation of magnetic forces, torques and stiffness," *IEEE Trans. Mag,* vol. 19, no. 6, Nov. 1983, pp. 2514-2519.