Shape Optimization of Dielectric Material Using Continuum Sensitivity and Adaptive Level Set Method

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Abstract—**This paper presents shape optimization of dielectric material using continuum sensitivity analysis and adaptive level set method. The velocity of shape deformation to optimize the system is calculated from result of the continuum sensitivity analysis. The shape is evolved by the adaptive level set method. This method matches the boundary data of element mesh with the zero level representing design variable, so that the efficiency and accuracy of the optimization are improved. Various numerical examples to optimize the shape of dielectrics are introduced.**

Index Terms—**Sensitivity analysis, Finite element methods, Numerical analysis, Design optimization**

I. INTRODUCTION

Shape optimization problem includes geometry change of analysis model. Such geometry change goes toward the direction to converge objective function of the problem. The direction could be determined from sensitivity analysis results. For mechanical and electromagnetic systems, the sensitivity analysis using the material derivative concept of continuum mechanics has been studied over the past few decades [1]-[3]. In this analysis, the accurate sensitivity can be calculated because the sensitivity formula is analytically derived. When the shape optimization problem is solved using the continuum sensitivity analysis with finite element method, certain numerical technique to evolve the shape is required. Many novel techniques have carried out the work. However, deformation of complicated shape with these techniques is quite difficult, since most of them need laborious discretization and surgical procedures.

Level set method is very versatile to express evolution of complicated geometry [4]. This method has been employed for shape and topology optimization problems years ago and some papers present the optimization in electromagnetic systems [5]. However, the level set method coupled with finite element modeling has an issue to be solved. That is discrepancy between the boundary of finite element mesh and the material interface which is zero level of level set function. If the material interface is design variable, the obscure materials near the interface cause inaccurate sensitivity analysis results. Consequently, the calculated shape deformation would not be exact. The adaptive level set method could relieve this problem. This method makes the material interface of zero level coincide with the boundary of finite element mesh.

This paper presents shape optimization using the continuum sensitivity and the adaptive level set method.

Especially, the shape of dielectrics in electrostatic system is optimized. The velocity of shape deformation is extracted from the sensitivity formula for the electrostatic system and it is applied to the level set equation. Several numerical examples are observed.

II. CONTINUUM SENSITIVITY ANALYSIS

In 2 dimensional electrostatic system, objective function of the optimization problem is generally defined as integration over specified region Ω or boundary Γ.

$$\mathrm{F}_1 = \int_\Omega g(V, \nabla V) d\Omega \text{ or } \mathrm{F}_2 = \int_\Gamma h(V, \nabla V) d\Gamma . \qquad (1)$$

where, V is the electric potential, g and h are the differentiable functions on the specified region and boundary, respectively. The sensitivity formula, which is the material derivative of the objective function, can be derived by using the concept of continuum mechanics. This formula usually has a form of boundary integration over the design variable.

$$\dot{\mathrm{F}} = \int_\gamma G(V, \lambda) \mathrm{V}_n d\Gamma . \qquad (2)$$

where, λ is the adjoint variable, V_n is the normal component of the velocity with respect to the design variable, and γ is the integration path on the design variable. The governing equation is considered as constraint and λ in (2) accounts for it. The tangential component of the velocity cannot contribute to the shape deformation, so that V_n is only important. Thus the velocity field is calculated as follows.

$$\mathrm{V}_n = kG(V, \lambda) . \qquad (3)$$

where, k is the scaling factor. The sign and magnitude of k are dependent on the direction of the objective function and the dimension of system, respectively.

III. ADAPTIVE LEVEL SET METHOD

When the level set method is used to solve problem with the shape deformation, level set function $\phi(\mathbf{x})$ distinguishes regions and their interfaces. The level set function is expressed in implicit form, and its isocontour of zero level represents the material interface which is design variable in this problem. The variation of the interface is defined by the following material derivative.

$$\frac{\partial \phi}{\partial t} + \mathbf{V} \cdot \nabla \phi = 0 . \tag{4}$$

where, t is the pseudo time and $\mathbf{V}$ is the velocity field which refers to desired direction and magnitude of the deformation. The result of continuum sensitivity analysis (3) is substituted to (4). This partial differential equation is called the level set equation. The equation is the Eulerian formulation capturing the interface by the implicit function, so that (4) easily expresses the evolution of the material interface.

Major difference between the adaptive level set method and the conventional one is treatment of the material interface. Fig. 1 shows the treatments of two methods. In the conventional method, zero level which represents the material interface and design variable is floating on the fixed mesh. Meanwhile, zero level coincides with the boundary of mesh in the adaptive level set method. The adaptive level set method may calculate more accurate shape deformation than the conventional one, since the sensitivity analysis results near the design variable is obvious. Moreover, the adaptive level set method can make the finite element fine near there, so efficient meshing is available.

IV. NUMERICAL EXAMPLES

The objective of numerical examples is to optimize the shape of dielectrics to maximize capacitance between electrodes. 2 dimensional electrostatic system is analyzed as the governing equation is solved by finite element method.

$$\varepsilon \nabla^2 V = -\rho . \tag{5}$$

where, ρ is the electric charge, and ε is the permittivity. Since electric voltage is applied as the source, the maximization of capacitance is equivalent to the maximization of system energy. The objective function is the system energy.

$$\mathrm{F} = \frac{1}{2} \int_\Omega \mathbf{E}(V) \cdot \mathbf{D}(V) d\Omega . \tag{6}$$

where, $\mathbf{E}$ is the electric field, and $\mathbf{D}$ is the electric flux density. The sensitivity formula of this problem is as follows.

$$\dot{\mathrm{F}} = \int_\gamma (\varepsilon^* - \varepsilon^{**}) \mathbf{E}(V^*) \cdot \mathbf{D}(V^*) d\Gamma . \tag{7}$$

where, * and ** mean both side regions of the material interface. The searching direction of this problem is toward increasing the objective function, thus the velocity field is calculated as follows.

$$\mathrm{V}_n = p(\varepsilon^* - \varepsilon^{**}) \mathbf{E}(V^*) \cdot \mathbf{D}(V^*) . \tag{8}$$

where, p is the positive constant. Its magnitude depends on

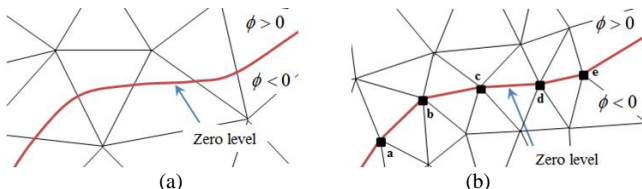(a)   (b)
Fig. 1. Treatments of material interface.
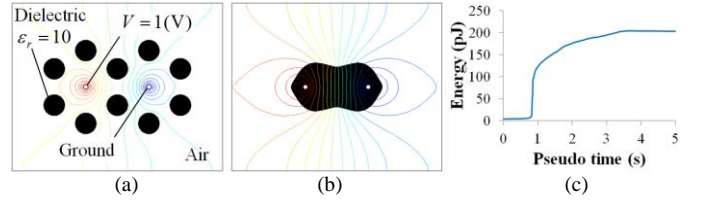(a) Conventional level set method. (b) Adaptive level set method.

(a)   (b)   (c)
Fig. 2. Shape optimization of dielectrics: 2 electrodes model.
(a) Initial design. (b) Optimum design. (c) System energy variation.

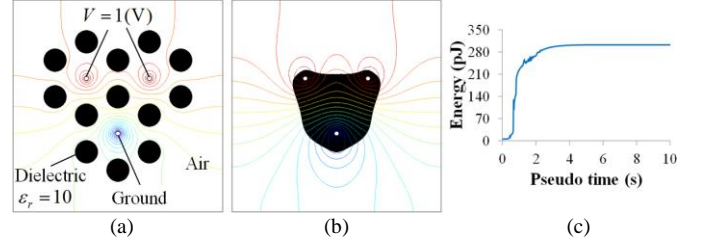(a)   (b)   (c)
Fig. 3. Shape optimization of dielectrics: 3 electrodes model.
(a) Initial design. (b) Optimum design. (c) System energy variation.

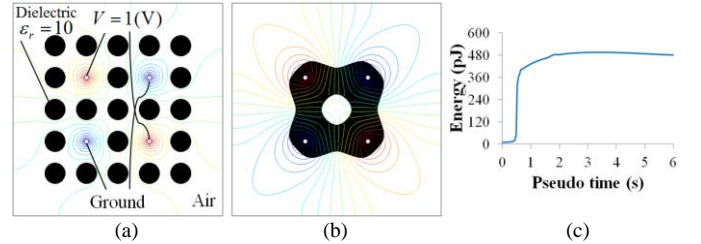(a)   (b)   (c)
Fig. 4. Shape optimization of dielectrics: 4 electrodes model.
(a) Initial design. (b) Optimum design. (c) System energy variation.

dimension of the system. The constraint of constant area must be satisfied when (8) is substituted to (4).

Fig. 2(a) shows shape and topology variations of 2 electrodes model. In the initial design, circular dielectrics are uniformly distributed. As pseudo time goes by, the circular dielectrics move to where electric field is strong. Finally, the dielectric is thick near the electrodes and forms concave shape between them. Fig. 2(b) shows the variation of system energy during optimization. The energy of optimum design is about 44 times larger than one of initial design. Fig. 3 and 4 show optimization results of 3 and 4 electrodes models, respectively.

Detailed contents in the shape optimization procedure and plenty examples will be presented in full paper.

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