Evolutionary algorithm-based multi-criteria optimization of triboelectrostatic separator

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***Abstract*—A device for electrostatic separation of triboelectrically charged plastic particles is modeled and optimized. Electric field in the system is solved numerically by a fully adaptive higher-order finite element method. The movement of particles in the device is determined by an adaptive Runge-Kutta-Fehlberg method. The shape optimization of the electrodes is carried out by a technique based on genetic algorithm NSGA-II and also on simulated annealing.**

***Index Terms*—Separation processes, triboelectricity, particle separators, finite element methods, optimization, genetic algorithms, simulated annealing**

I. Introduction

Nowadays, an intensive research aimed at the possibilities of recycling plastic materials is conducted worldwide because in a lot of applications the recycled materials may well replace the new ones. The necessary prerequisite of this reprocessing is a high-quality separation of particular kinds of plastics grinded into small sphere-like particles. One of the advanced techniques of separation of such particles is based on the triboelectric effect [1]. It is known that when electrically non-conducting particles of two different levels come into contact with electric charge, one of them becomes more positive (or negative) with respect to the other one. And their trajectories in electric field (affected by the charge they carry) may be quite different. This is clear from Fig. 1 showing a scheme of the separator to be analyzed and optimized.

The separator consists of two electrodes, one of them being grounded. The voltage of the other electrode is on the order of tens kV. The electrodes may be covered by insulating material that prevents the particles from recharging in case of the direct impact with them. At the bottom of the device there are several recycle bins used for accumulating of particular levels of plastics. One of the principal demands is to optimize the shape of the electrodes and widths of the bins so that the particles of different kinds fall down exactly to the corresponding bins.

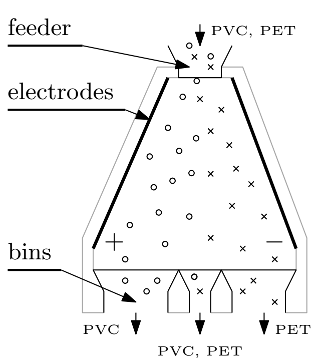

Figure 1: Basic arrangement of discussed separator

This kind of separation was analyzed and evaluated by several research teams. Some of them [2] were aimed at a detailed description of the technology and its practical applications. Modeling of the particle trajectories (with the aim to appropriately design their arrangement) based on classical finite difference and finite element algorithms can be found for example in [3]. But the accuracy of the results was not high because the distribution of nonuniform electric field and motion of the particles were calculated separately and the computations were based on low-order mapping methods. The problem of the shape optimization was mostly solved empirically, starting from the comparison of several different arrangements, without applying direct optimization techniques.

The paper represents an attempt to fill in the above gaps. The distribution of electric field between the electrodes is determined using a fully adaptive higher-order finite element method, i.e., with much higher accuracy than that provided by the mentioned low-order method. The movement of the particles affected by electric field, gravity and drag aerodynamic resistance is modeled by an adaptive Runge-Kutta-Fehlberg method with a time-varying time step. The shape optimization of the electrodes is carried out using a technique based on the genetic algorithm NSGA-II and on the simulated annealing algorithm.

II. Mathematical model and its numerical solution

At this stage of research, the problem of determining the trajectories of the particles is solved on the assumption of neglecting the mutual Coulomb forces acting between the individual particles (which are, however, small with respect to the force exerted by the external electric field). The distribution of electric field $\mathbf{E}$ within the separator follows from the equation

$$\nabla \cdot (\varepsilon \nabla \varphi) = 0 \,.$$

The trajectory of a particle is then described by the equations

$$m \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{F}_e + \mathbf{F}_g + \mathbf{F}_a \,, \;\; \mathbf{v} = \frac{\mathrm{d}\mathbf{s}}{\mathrm{d}t} \,,$$

where

$$\mathbf{F}_e = Q\mathbf{E} = -Q \cdot \nabla\varphi\,,$$

$$\mathbf{F}_g = m\mathbf{g}\,,\ \mathbf{F}_a = -\mathbf{v}\frac{1}{2}\rho c S v\,.$$

Here, ε denotes the relative permittivity, φ is the electric potential, m stands for the mass of the particle, Q is its charge, $\mathbf{v}$ denotes its velocity, $\mathbf{s}$ is the trajectory, ρ represents the mass density of air, c is the coefficient of friction, S denotes the cross section of the particle, and, finally, $\mathbf{F}_e$ is the electric force, $\mathbf{F}_g$ stands for the gravitational force and $\mathbf{F}_a$ denotes the drag aerodynamic force.

The field computations are carried out by our own code Agros2d [4] based on a fully adaptive higher-order finite element method. The code closely cooperates with the library Hermes [5] containing the most advanced procedures for the numerical solution of partial differential equations.

III. Shape optimization

Two functionals were used for the optimization. The first one is the voltage U between the electrodes that should be as low as possible, the second one F is given by the distance of fall-down of the particle from the corresponding bin. The computations were performed for a sample of 500 particles whose charges and radii were subject to the Gauss distribution and their initial positions in the feeder followed uniform distribution.

Since no gradient information is directly available, two heuristic algorithms were used for the optimization of the problem. Simulated annealing (SA) fixes the voltage U between the electrodes and for the fixed value of voltage it minimizes the penalization F. This algorithm is a generalization of a greedy algorithm with the difference that there is a positive probability of entering a state with worse objective value. This probability decreases in time, allowing the algorithm to explore the feasible region at the beginning, and basically becomes the greedy algorithm near the end. With a fixed probability, this step is really random and with the complementary probability it depends on the problem. This enabled the simulated annealing to converge well in $\mathbb{R}^{11}$ after only about 300 evaluations.

On the contrary, the genetic algorithm NSGA-II works with a population of feasible solutions and instead of fixing the voltage it compares the feasible solutions on the base of non-dominance, enabling the creation of the whole Pareto front in only one run. A modification was added to the algorithm, enabling to mutate not only children, but also parents. This caused the child population to be greater but since the next population is chosen from the union of parent and child populations, this is not a problem. This parent mutation was chosen to be problem-dependent, which enabled faster convergence in comparison with the standard algorithm.

IV. Results and their discussion

For illustration, Fig. 2 shows the comparison of the Pareto fronts obtained by both mentioned algorithms. Figure 3 then shows the optimized shapes of the electrodes (they are not

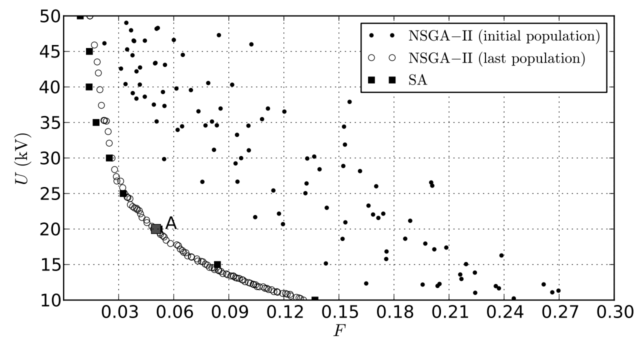

Figure 2: Comparison of Pareto front obtained by NSGA-II and SA algorithms (point A shows selected variant in Fig. 3)

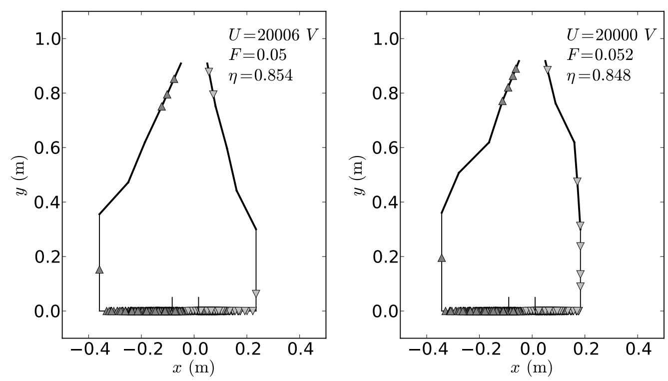

Figure 3: Selected variants obtained by NSGA-II algorithm (left) and SA (right), markers show final positions of separated particles and η denotes efficiency of separation process

symmetric because the average charges of both levels of plastic particles differ one from another). At present, a laboratory device is being built for experimental verification of the most important results.

V. Acknowledgment

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