

# Reliability-Based Optimum Tolerance Design for Industrial Electromagnetic Devices

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**Abstract**—Tolerance design to find the best compromise between manufacturing price and quality is being extensively researched because mass production with higher quality at lower cost is significantly required. However, current tolerance designs are improper for the products such as electromagnetic devices that demand the high reliability, i.e. the very small failure rate. In this paper, new tolerance design problem with high reliability is formulated. And reliability-based tolerance design is proposed in order to obtain high reliable products despite manufacturing tolerances. The proposed method quantifies the reliability using reliability analysis, which is reflected to tolerance design. To validate the proposed method, tolerance design is applied to IPM motor that contains tolerances of PM.

**Index Terms**— Electromagnetic devices, Optimization method, Permanent magnet motors, Reliability, Tolerance analysis.

## I. INTRODUCTION

In general, from the manufacturing point of view, tolerances of industrial products are inherent and inevitable, which gives rise to defective products. Design engineers would like to assign narrowed tolerances for quality of products, but manufacturers usually prefer to deal with wider tolerance due to cost and production difficulty. Thus, designers have to tread the tricky path between maintaining quality and keeping manufacturing prices down through tolerance design.

Tolerance design technique to find the best compromise between manufacturing price and quality is being extensively researched. This study provides a trade-off relationship to link the elements of quality loss expressed by design tolerance and manufacturing cost. Many researchers pay attention to the allocation of design tolerances for minimum spending. In order to produce robust products, Taguchi proposed an overall quality control system. In his books, the quality of a product is defined as the loss incurred due to the deviations of the products' characteristics from their target values. With the assistance of loss functions, several researchers redefined the relationship between quality loss and manufacturing cost in one equation. However, these methods cannot provide design solution but only the direction to the optimum solution [1]. Thus, previous tolerance design techniques are improper for the products such as electromagnetic devices that demand high reliability, i.e. small failure rate.

In this paper, new tolerance design problem with high reliability is formulated. And reliability-based tolerance design is proposed in order to obtain high reliable products while maximizing manufacturing tolerances. The proposed method

quantifies reliability using reliability analysis, which is reflected to optimization of tolerance design.

To validate proposed method, tolerance design of released interior permanent magnet (IPM) motor that contains tolerances of permanent magnet is performed. The failure rate of IPM motor is defined as a specified value of back EMF influenced significantly by the tolerances of PM. Therefore, it is necessary to employ the tolerance design optimization in which failure rate of products is maintained within the required reliability while maximizing the tolerances of PM. Quantification of the reliability of IPM motor is estimated based on 5,232 data of IPM motors made by Keyang Electric Machinery, Korea.

## II. RELIABILITY-BASED TOLERANCE DESIGN

A quantitative measurement of economic loss proposed by Taguchi has provided an essential tool to evaluate the quality of the products. This method can provide a solution with robust performance by using quality loss function but it cannot consider failure probability which doesn't contain requirement. Also, it is unable to control allowable tolerances because its measurement has no relation to tolerance range. In this paper, by using mathematical relationship between cost and tolerance as objective function, demanded tolerance range for quality can be ensured in low cost. We use the inverse power machining cost-tolerance model [2] given by

$$f(\delta) = c_0 \frac{1}{\delta^{c_1}} \quad (1)$$

where  $\delta$  is the range of tolerance and  $c_0$  and  $c_1$  are model parameters usually determined by curve fitting empirical data for a particular machining process.

Compared to other tolerance design techniques, proposed method can be conducted under the condition requiring failure rate since reliability analysis is performing simultaneously with tolerance design. Formulation of reliability based tolerance design can be expressed as

$$\begin{aligned} & \text{Find} \quad \boldsymbol{\mu}_{\text{tolerance}}, \boldsymbol{\sigma}_{\text{tolerance}} \\ & \text{Minimize} \quad \text{cost} = f(\boldsymbol{\sigma}) = \sum_{i=1}^n \left( c_0 \frac{1}{\sigma_i^{c_1}} \right) \end{aligned} \quad (2)$$

$$\text{Subject to} \quad G_j = P\{g_j(\boldsymbol{\mu}, \boldsymbol{\sigma}) \leq g_{\text{required}}\} \leq \text{required failure rate}$$

Mean and deviation of tolerances are selected as design variables. As mentioned above, we use cost-tolerance model as an objective function and failure rate based reliability analysis

as constraint functions. The reliability analysis technique will be discussed in the next chapter.

### III. AKAIKE INFORMATION CRITERIA

The probability of a system to satisfy the design requirement under tolerances is referred to as *reliability*. In this paper, we select Akaike information criteria (AIC) method that determines the best estimated distribution for discrete data and calculate system's reliability from the selected distribution [3]. For performing tolerance design optimization considering reliability efficiently, we have to select robust sampling technique with less variability from sample to sample. Hence, latin hypercube design producing more stable statistical information is used in this research.

### IV. OPTIMUM TOLERANCE DESIGN OF IPM MOTOR

#### A. Analysis model

The motor selected in this paper is used for electric sub-water pump of hybrid vehicle. Rated power and rated speed are 150W and 3000rpm. The detail specifications of this motor are listed in Table 1. This motor is driven by rectangular voltage waveforms coupled with the given rotor position.

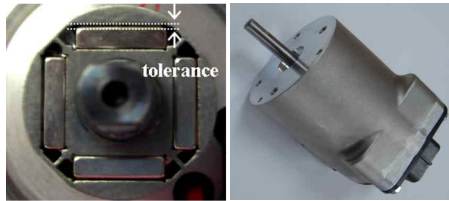


Fig. 1. Cross and side section view of fabricated rotor

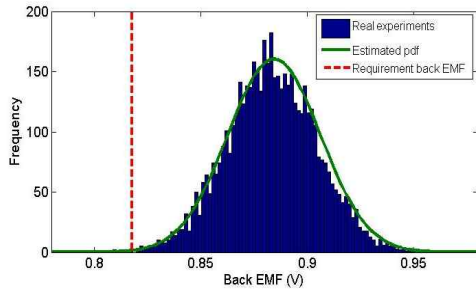


Fig. 2. Histogram obtained from real experiments and industrial products

TABLE I  
MAIN SPECIFICATIONS AND REQUIREMENTS OF IPMSM

	Value	Note
Type	IPMSM	-
Slot	6	-
Power	150 W	@3200rpm
DC link voltage	12 Vdc	-
Current limit	15 Arms	-
Br	1.273 T	-

TABLE II  
RESULT OF RELIABILITY-BASED OPTIMUM TOLERANCE DESIGN IN IPMSM

	Design variable	Cost	Failure rate(%)
Present design	[0.5, 0.75, 0.5, 0.75]	-0.06	[0.18, 0.01, 0.01]
Optimum design	[0.19, 0.77, 0.34, 1]	-0.073	[0.08, 0.1, 0.1]

#### B. Reliability analysis

Tolerances can break out any parts of the motor but tolerances of PM are especially the decisive factor in variation of performances. In the manufacturing process, irregularly coated PM is glued into the interior of the rotor rather than being pressed. So, thickness of coating and space between rotor and PM are a tolerance to be reckoned with in performance of motor as shown in Fig. 1. In this study, we use a test result of tolerances based on IPM motors made by Keyang Electric Machinery. The real experiment data from industrial products and probability density function (PDF) estimated by AIC method are shown in Fig. 2.

#### C. Reliability-based optimum tolerance design in IPM motor

Formulation of reliability based tolerance design in IPM motor can be expressed as

$$\begin{aligned}
 & \text{Find} \quad \boldsymbol{\mu}_{\text{tolerance}}, \boldsymbol{\sigma}_{\text{tolerance}} \\
 & \text{Minimize} \quad \text{cost} = f(\boldsymbol{\sigma}) = \sum_{i=1}^2 \left( c_0 \frac{1}{\sigma_i} \right) \\
 & \text{Subject to} \quad G_1 = P\{g_1(\boldsymbol{\mu}, \boldsymbol{\sigma}) \leq E_{\text{requirement}}\} \leq 0.001 \\
 & \quad \quad \quad G_{2,3} = P\{g_{2,3}(\boldsymbol{\mu}, \boldsymbol{\sigma}) \leq \delta_{\text{coated}}\} \leq 0.001 \\
 & \text{Where} \quad \boldsymbol{\mu}_0 = [0.5, 0.5], \boldsymbol{\sigma}_0 = [0.75, 0.75] \\
 & \quad \quad \quad [0, 0] \leq \boldsymbol{\mu} \leq [1, 1], [0, 0] \leq \boldsymbol{\sigma} \leq [1, 1]
 \end{aligned} \tag{3}$$

Mean of thickness and width and deviation of those are selected as design variables. Required failure rate of back EMF and allowable coated space in PM are treated as design constraints. Demanded failure rate is 0.1%, which is smaller than currently released motor, 0.18%.

#### D. Result

Finally, we can find an optimum as given in Table II. The objective function, i.e. manufacturing cost value, is decreased by 21.7% compared with present products. Also the number of defective motors reduces from 18 to 8 due to the reliability-based constraints as shown in Table II.

### V. CONCLUSION

In this paper, we propose optimum tolerance design technique that tolerances of design variables are maximized while satisfying specified failure rate. Quantifications of tolerances and failure rate are estimated based on sampled data obtained from mass production. Through the proposed method for IPM motor, we can obtain the new design which has larger tolerance than current controlled tolerance as well as lower failure rate.

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