

Internally Consistent Nonlinear Behavioural Model of a PM Synchronous Machine for Hardware-in-the-Loop simulation

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Abstract—Hardware-in-the-Loop (HIL) simulation allows hardware implementations of some components of a system to be tested in conjunction with other components which are only emulated based on behavioural models. For example, a physical prototype of an Engine Control Unit for a hybrid vehicle can be tested with virtual prototypes of the motor and vehicle. This paper presents a new method to obtain realistic behavioural models of Permanent Magnet Synchronous Machines from Finite Element Analysis. The new method guarantees that the model will be internally consistent, for example it will not violate the principle of conservation of energy.

Index Terms—Electric machines, Brushless machines, Permanent magnet machines, Rotating machines, Brushless motors, Electric motors, Permanent magnet motors, Circuit simulation, Numerical simulation, Finite element methods, Spline, Interpolation.

I. INTRODUCTION

The increasing complexity of motor applications and increasingly shortened design cycles has created a need for the ability to test some components of a system before other parts of the system have been completed. To fill this need, specialized Hardware-in-the-Loop (HIL) simulation tools, such as eDRIVESimTM from Opal-RT, are available which use dedicated hardware to simulate a Permanent Magnet Synchronous Machine (PMSM) in real-time (< 1 μ s time step). The nonlinear models hardcoded into the simulator are based on tables of flux, inductance and torque as a function of rotor position and winding currents.

For realistic models these tables must be obtained through finite element analysis (FEA). As with similar approaches to constructing behavioural models of PMSMs [1] and [2], these tables can be obtained directly from FEA through field integration. However, imperfect FEA solutions can lead to models which are not internally consistent and which violate the principle of conservation of energy. Some models are inherently consistent but inappropriate for table calculation, for example those based on magnetic circuits such as [3] (which is not nonlinear), and [4] (which cannot readily be adapted to create the nonlinear tables). This paper describes how a model based on coenergy can be used to create tables which are internally consistent.

II. HIL SIMULATION

A. Update equations

The behavioural model of a PMSM, neglecting eddy currents and iron losses, can be described in two equations. Because these equations must be implemented in hardware and run at sub-microsecond time steps, they are written in the form of update equations for a time step rather than as differential equations. The first is an update equation for the winding currents based on terminal voltages [5]:

$$\mathbf{I}(t + \Delta t) = \mathbf{I}(t) + \mathbf{L}_{inc}(\theta, \mathbf{I})^{-1} \left(\int_{t=0}^{t+\Delta t} \mathbf{V}(t) - R\mathbf{I}(t) dt - \mathbf{\Phi}(\theta, \mathbf{I}) \right) \quad (1)$$

where θ is the rotor position, \mathbf{I} is the current vector, \mathbf{V} is the voltage vector, R is the phase resistance, $\mathbf{\Phi}(\theta, \mathbf{I})$ is the flux linkage vector, and $\mathbf{L}_{inc}(\theta, \mathbf{I})$ is the incremental (a.k.a. differential) inductance.

The second equation relates the magnetic torque $T_m(\theta, \mathbf{I})$ to the mechanical dynamics of the load and is specific to the application.

B. Straightforward table calculation from FEA

The functions of current and position in (1) are based on tables of flux, incremental inductance, and torque. The tables of flux and torque can be obtained directly from FEA. The incremental inductance is the derivative of the flux and can be calculated by constructing an interpolating function for the flux and taking its derivative or by finite difference of the flux table values.

Good results can be and have been obtained with this straightforward approach [5]. However, due to discretization errors, incomplete convergence, and numerical noise in the FEA solutions, obtaining the tables this way can lead to models which are not internally consistent. This internal inconsistency can take several forms:

- violation of energy conservation
- cogging torque which does not integrate to zero over one rotor revolution
- an incremental inductance matrix which is not symmetric

To repair these inconsistencies the following approach is proposed.

III. INTERNALLY CONSISTENT MODEL

The problem of representing a physical system with a consistent model occurs in many disciplines. The requirement for an effective and physically sound model of the B-H relationship for an anisotropic magnetic material motivated a model based on coenergy [6]. The same approach can be applied to this model. The functions of position and current on the right hand side of (1), as well as the torque, can also be obtained from the coenergy W_m [7]:

$$\Phi(\theta, \mathbf{I}) = \frac{\partial W_m(\theta, \mathbf{I})}{\partial \mathbf{I}} \quad (2)$$

$$\mathbf{L}_{inc}(\theta, \mathbf{I}) = \frac{\partial \Phi(\theta, \mathbf{I})}{\partial \mathbf{I}} = \frac{\partial^2 W_m(\theta, \mathbf{I})}{\partial^2 \mathbf{I}} \quad (3)$$

$$T_m(\theta, \mathbf{I}) = \frac{\partial W_m(\theta, \mathbf{I})}{\partial \theta} \quad (4)$$

A model based on the coenergy function will, by construction, have none of the internal inconsistencies itemized in the previous section. However obtaining an accurate model of the coenergy from the tabulated FEA data is a nontrivial task.

A. Direct coenergy model

The direct approach would be to obtain a table of coenergy values by integration of the FEA fields, and interpolate in this table using Fourier series and cubic splines. This approach would lead to a consistent model, but would have issues with accuracy, since taking a derivative is inherently unstable and amplifies numerical noise. The error becomes particularly unacceptable for $\mathbf{L}_{inc}(\theta, \mathbf{I})$, since it is a second derivative.

B. Higher-order coenergy model

A smoother, more accurate, and consistent coenergy model can be obtained in the following manner:

S-1 Construct the tables for flux linkages as a function of position, current magnitude and advance angle by field integration of FEA solutions on a regular grid.

S-2 Take the dot product of flux linkage from S-1 with current and divide by current magnitude I_p to obtain the derivative of coenergy with respect to current magnitude:

$$\frac{\partial W_m}{\partial I_p} = \frac{\mathbf{I} \cdot \Phi(\theta, \mathbf{I})}{I_p} \quad (5)$$

S-3 At each position and advance angle construct a cubic spline to interpolate $\partial W_m / \partial I_p$ as a function of current magnitude.

S-4 Integrate the cubic spline with respect to current magnitude to obtain a fourth-order spline of the coenergy.

This approach results in a model for the coenergy with C3 continuity, so even $\mathbf{L}_{inc}(\theta, \mathbf{I})$ will have C1 continuity. The model is not quite complete. Step S-4 above only determines the coenergy within a constant of integration. This "constant" is the coenergy at zero current, and is a function of rotor position.

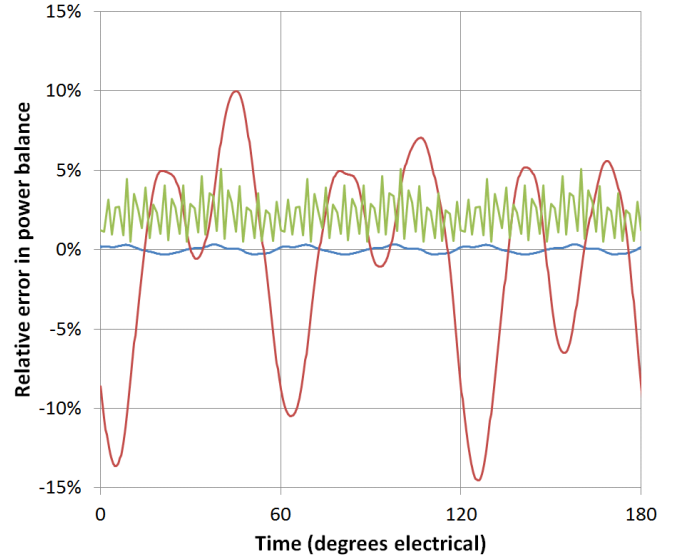


Figure 1: Error in power balance for time-stepping FEA (green curve), "straightforward" model (red curve) and the internally consistent model of section III-B (blue curve).

IV. COMPARISON WITH STRAIGHTFORWARD MODEL

The proposed model can be validated by comparison with transient FEA. It can also be compared to the "straightforward" model described in the introduction, which interpolates the flux and torque directly in the tables of the same quantities obtained from FEA. For these comparisons the motor used is the 2004 Toyota Prius hybrid vehicle drive motor [8]. The motor is current driven at 4000 rpm with 100 A rms at an advance angle of 45° yielding an output of 107 kW. As shown in Fig. 1, the internally consistent model of section III-B has a power imbalance of less than 0.5%, while the "straightforward" model has errors of more than 10%.

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