# Nonlinear Dynamic Characteristic Analysis of Linear Actuator for Compressor

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*Abstract***—This paper researches the nonlinear dynamic characteristic analysis of the linear actuator for the compressor. For the Nonlinear modeling, the electrical and mechanical parts of the linear actuator considering the nonlinearity are mathematically modeled. The nonlinear equivalent circuit and equation are presented by the mathematical modeling. The nonlinear magnetic flux linkage and thrust force according to the input current and the mover position are obtained from the finite element analysis (FEA) method. In order to be applied to the equivalent equation, the nonlinear magnetic flux linkage and nonlinear thrust force look-up table are configured by the magnetic analysis result value. In addition, the linear and nonlinear characteristics according to the series resonant PWM inverter are analyzed. The nonlinear characteristic analysis of the linear actuator is verified through experiments.**

**inverter, Magnetic flux leakage, Compressor**

# I. INTRODUCTION

In recent years, the high performance linear actuator for the compressor is being studied due to ozone depletion caused by refrigerant. A linear actuator used as a mechanism for compressing the expended refrigerant gas in a cooling apparatus such as a refrigerator or air-conditioner has been developed [1]-[4]. The conventional characteristic analysis of linear actuator is focused on the coupled modeling of the<br>electrical and  $k_x(i, x)$  refer to the nonlinear value for<br>electrical and mochanical [5]. However, the phase current electrical and mechanical [5]. However, the phase current distortion and position error of mover when driving linear actuator occurs because it has nonlinear inductance and nonlinear thrust force depending on the input current. 1. INTRODUCTION<br>
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To solve this problem, the electromagnetic nonlinear analysis and nonlinear modeling of linear actuator studied in this paper. The specifications of the linear actuator are given in Table 1.

TABLE I SPECIFICATIONS OF LINEAR ACTUATOR

			III. ELECTROMAGNETIC ANALYSIS OF LINEAR ACTUATOR
Item	Unit	Value	
Input current		$-1.65 \sim 1.65$	The magnetic Flux linkage of the linear actuator has the
Current density	$A/mm^2$	1.52	nonlinearity depending on the input current and position of the
Winding type		Concentrated	mover. The input current according to the mover positions is
Number of pole	Pole		supplied in the linear actuator using the finite element analysis
Winding number of Coil	Turn	515	method. Fig. 1 show respectively magnetic flux lines and
Inductance	mH	220	
Winding resistance		13.9	density according to the input current and mover position.
Length of mover	mm	18	Fig. 2 and Fig. 3 show respectively the flux linkage and
stroke of mover	mm	16.3	thrust force according to the input current and mover positions

In this paper, special considerations of some aspects with respect to the nonlinear characteristic analysis of the considered linear actuator are discussed. This study is essential since in addition to linear and nonlinear dynamic characteristic comparison, also the practical, characteristic analysis with respect to driving the series resonant PWM inverter have to be taken into account.

### II. MATHEMATICAL MODELING OF LINEAR ACTUATOR

The governing equations of the linear actuator are presented by electrical and mechanical equation. The nonlinear electrical equation is given by the following (1) [5].

$$
V = Ri + k_i(x, i)\frac{di}{dt} + k_x(x, i)\frac{dx}{dt} + \frac{1}{C}\int i\,dt\tag{1}
$$

*Index Terms*—Linear actuator, Nonlinear equation, Resonant  $x$  is the displacement of position,  $k_i(x, i)$  is flux variations **EXECT:** Analysis of<br>
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mechanical equation. The n Where *V* , *R* , and *i* are the voltage, resistance and current, *XOT* COMPIPTESSOT<br> *x* ee and Yun-Hyun Cho<br> *x* alab, Dong-A University<br>
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energet displacement.

The mechanical equation given by the following (2)

$$
k_x(i, x)i = M\frac{d^2x}{dt} + (C_f + C_g)\frac{dx}{dt} + (K_m + K_g)x
$$
 (2)

addition to linear and nonlinear dynamic characteristic<br>son, also the practical, characteristic analysis with<br>o driving the series resonant PWM inverter have to be<br>o account.<br>MATHEMATICAL MODELING OF LINEAR ACTUATOR<br>gover addition to linear and nonlinear dynamic characteristic<br>
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equations of the linear actuator are<br>
and mechanical equatio Where *M* is mass,  $C_f$  is friction constant,  $C_g$  is gas damping constant,  $K_m$  is gas spring constant and  $K_g$  is gas II. MATHEMATICAL MODELING OF LINEAR ACTUATOR<br>
The governing equations of the linear actuator are<br>
presented by electrical and mechanical equation. The nonlinear<br>
electrical equation is given by the following (1) [5].<br>  $V =$ converting mechanical force into electrical force.  $V = Ri + k_i(x, i) \frac{di}{dt} + k_x(x, i) \frac{dx}{dt} + \frac{1}{C} \int idt$  (1)<br>
re  $V$ ,  $R$ , and *i* are the voltage, resistance and current,<br>
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here *V*, *R*, and *i* are the voltage, resistance and current,<br>
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ight side is the<br>  $\frac{m + K_g}{\omega}$ )  $j$  (3)<br>  $\frac{m + K_g}{\omega}$ )  $j$ <br>
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The left and right si *V* =  $Ri + k_i(x, i) \frac{di}{dt} + k_x(x, i) \frac{dx}{dt} + \frac{1}{C} \int idt$  (1)<br>
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spring constant and

The nonlinear equivalent equation can be expressed as shown in (3) from (1) and (2). The left and right side is the electrical and mechanical equivalent equation.

$$
V = [R + (k_i(x, i) - \frac{1}{\omega C})j]i + [\frac{k_x^2(i, x)}{C_f + C_g + (m\omega - \frac{K_m + K_g}{\omega})j}]i \tag{3}
$$

## III. ELECTROMAGNETIC ANALYSIS OF LINEAR ACTUATOR

Length of mover  $\frac{mm}{mm}$  18 Fig. 2 and Fig. 3 show respectively the flux linkage and stroke of mover *mm* 16.3 thrust force according to the input current and mover positions. The input current range is  $-3A \sim 3A$  and moving distance range is  $-8.1$ mm  $\sim 8.1$ mm.



Fig. 1. Magnetic flux lines and density according to mover position



Fig. 2. Flux linkage according to input current and mover position



Fig. 3. Thrust Force according to input current and mover position

# IV. SIMULATION RESULT

The simulation results present the linear and nonlinear characteristics when driving the series resonant PWM inverter.

Fig. 4 shows the nonlinear and linear input voltage. The nonlinear voltage RMS value is 4V greater than linear voltage.

Fig. 5 shows the nonlinear and linear input current. The nonlinear current leads 26.3° more than the linear current in the negative direction.

Fig. 6 shows the estimated nonlinear and linear inductance. The linear inductance value is constant at 220mH. In contrast, the nonlinear value is nonlinear.

Fig.7 shows the estimated nonlinear and linear thrust force. The peak to peak value of linear thrust force is 127.46Nmm/s. In contrast, the nonlinear value is 141.77Nmm/s and leads 26.3° more than linear value in the negative direction.



Fig. 4. Nonlinear and linear input voltage



#### V. CONCLUSION

The nonlinear dynamic characteristic analysis of the linear actuator according to the steady and transient state of driving the linear actuator will be studied and verified through experiments.

## ACKNOWLEDGMENT

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