On the geometric uncertainties of an electrical machine: stochastic modeling and impact on the performances

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Abstract —**Mass production processes are generally sources of variability on the geometry of the product. A probabilistic approach is one way to take into account this variability and to evaluate its effect on the performances of the product. In this communication, we apply this approach to study the effect of the variability on dimensions of the stator on the performances of an electrical machine.**

I. INTRODUCTION

In the field of electrical machine modeling, numerical approaches are more and more widely used. With the performance increasing of the computational tools, it is now possible to simulate, in a reasonable time, electromagnetic devices with a complex geometry and with more and more realistic material behavior law. In these models, the input parameters, the geometry (dimensions…), the characteristics of materials (permeabilities…) and the sources (current densities…), are assumed to be perfectly known. In practice, the values of the input parameters are not generally well known and are often tainted by uncertainties. For example, dimensions are subject to deviations and are generally given in a tolerance interval. These uncertainties on the input data lead to uncertainties on the outputs of the numerical model. Since the numerical models are more and more accurate, these uncertainties on the outputs are sometimes more significant than the modeling errors and numerical errors and should therefore be taken into account. To model and to propagate uncertainties on the input data, stochastic approach can be used which is currently presented in numerous studies. This approach consists in modeling the uncertain inputs and outputs by random variables or fields [1, 2]. This approach requires the characterization and the modeling of the input data as random variables and then the numerical solution the stochastic model. In the computational electromagnetics field, numerous studies address the second step [3, 4] but few address both steps at the same time.

In this paper, we propose to study the influence of the uncertainties bore by the dimensions of the stator on the performances of a synchronous machine. In the first part, we present the model chosen to describe the uncertainties bore by the dimensions. Then, we quantify the effect of these uncertainties on the torque at no load of the electrical machine. The electrical machine is modeled using the Finite Element Method and an approximation based on a polynomial chaos expansion is used to quantify the uncertainties on the torque.

II. DESCRIPTION OF THE STATOR

We are interested in a stator of an electrical machine presented in the Fig. 1. We will focus our study on the uncertainties of the radius of the 36 teeth. We assume that the other dimensions of the stator are deterministic and are equal to their nominal value. We perform measurements on 5 stators. The interior surface of the stator is divided into 30 layers. For each layer, the coordinates representing the position of 36 points corresponding to 36 teeth are determined by measurement (Fig. 2). We obtain then 30x36 =1080 coordinates. According to these coordinates, we can define the center axe (represented by the point O in Fig. 1) and then the 1080 radii corresponding to the 1080 points.
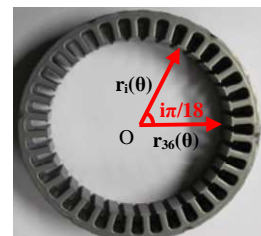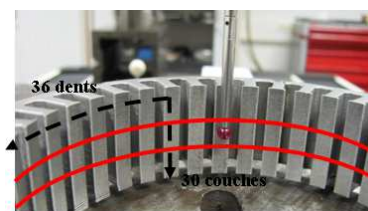

Fig. 1. Studied stator

Fig. 2. Measurement.

The histogram of the radius is presented in the Fig. 3 for the 5 stators. The vertical purple lines represent the tolerance interval. One can notice that some values are located outside the tolerance interval.
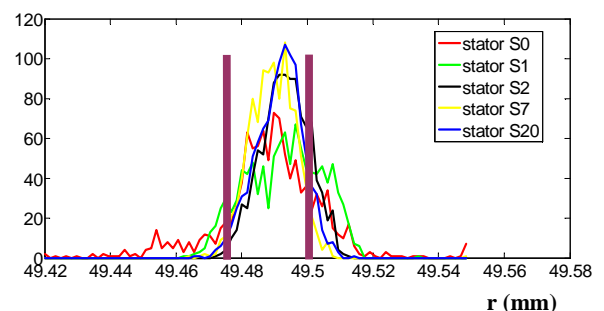

Fig. 3. Histogram of the radius

III. PROBABILISTIC MODELING

One can notice that the interior surface of the stator can be *a priori* represented by 1080 radius. A stochastic model of 1080 random variables can be used to model the variability of the interior surface of the stator. However, a model with such number of input random variables can lead to an excessive

numerical cost while applying an uncertainties propagation method. Furthermore, the 1080 random variables can be strongly correlated. A reduced model could be then interesting.

To establish such reduced model, principal component analysis (PCA) can be used [5]. This method consists in representing the 1080 random variables by N mutually uncorrelated random variables with N < 1080. However, due to its unphysical nature, the reduced model obtained by this method can give results that can be difficult to interpret. In fact, the new random parameters can not be linked simply to the geometry anymore. In [6], we have proposed to represent the deformation of the stator using a discrete Fourier transformation. After an analysis based on the measurement on the 5 stators, the main harmonic of the deformation of the stator has been determined. Finally, the model retained to describe the interior surface of the stator is written under the form:

$$r_{ij}(\theta) \approx R_1 + \tau_{0j}(\theta) + \tau_{2j}(\theta)\cos\left[2\frac{2\pi i}{36} + \varphi_{2j}(\theta)\right] +$$
$$\tau_{4j}(\theta)\cos\left[4\frac{2\pi i}{36} + \varphi_{4j}(\theta)\right] + \tau_{6j}(\theta)\cos\left[6\frac{2\pi i}{36} + \varphi_{6j}(\theta)\right] \quad (1)$$

with $r_{ij}(\theta)$ the radius of the tooth i related to the jth layer, R_1 the nominal value of radius, $\tau_{0j}(\theta), \tau_{2j}(\theta), \varphi_{2j}(\theta), \tau_{4j}(\theta), \varphi_{4j}(\theta), \tau_{6j}(\theta)$ and $\varphi_{6j}(\theta)$ the random variables related to the most significant harmonic (2, 4 and 6). In our case, the number of realizations of $\tau_{ij}(\theta)$ is very small and equal to 5. To model them, we assume that the random variables are independent and gaussian. The mean value and the standard deviation have been estimated from the available measurements. In Table 1, we have reported the mean and the standard deviation for the layer j =15. We can see that the modes 0, 2, 4 and 6 have the same order of variability.

TABLE I
MEAN VALUE AND STANDARD DEVIATION OF RANDOM
VARIABLES τ_k ASSOCIATED TO THE LAYER 15

Mode	τ_0	τ_2	τ_4	τ_6
Mean (μm)	1.93	5.16	1,77	3.74
Std (μm)	3.86	3.93	1.18	1.59

IV. IMPACT ON THE MACHINE PERFORMANCES

We consider now a 2D synchronous machine with a wounded rotor supplied by a current I. We want to evaluate the effect on the torque at no load (cogging torque) of the uncertainties on the tooth radii of the stator modeled previously. We assume that geometric uncertainties are bore only by 7 random variables $\tau_0(\theta), \tau_2(\theta), \tau_4(\theta), \tau_6(\theta), \varphi_2(\theta), \varphi_4(\theta), \varphi_6(\theta)$ characterized previously. The other dimensions are considered deterministic. The machine is represented using a Finite Element Model. The uncertainties on the geometry are taken into account using a transformation Method [3]. A Polynomial Chaos Expansion (PCE) is used to approximate the random RMS value of the torque [1]. The coefficients of the PCE are determined using a projection technique. First, we have studied the effect of each harmonic independently assuming that the other harmonics are constant equal to their mean value. The mean value and the standard deviation of the RMS value of the torque are reported in Table 2. First, we can notice that the contribution of each harmonic to the variability of the RMS value of the torque is very small and less than 1.2%. Even though, the variabilities of the harmonics of the deformation of the stator are of the same order, we can see also the harmonic 0 has the most significant effect. The variability induced by the harmonics 2, 4 and 6 are almost negligible due to an auto compensation effect of the force distribution inside the machine.

A sensitivity analysis has been undertaken by calculating the Sobol indices [7]. It confirms that the most influent variable is the random parameter $\tau_0(\theta)$. The contribution of the magnitude $\tau_2(\theta)$, $\tau_4(\theta)$, $\tau_6(\theta)$ represents less than one per cent. The contribution of the phase is almost negligible as well as the joint effect of the input parameters.

Globally, this study illustrate that even though we have radius values outside the tolerance interval, the influence on the torque is almost negligible. If we had considered that the radius of each tooth were equal to the minimum and the maximum values of the tolerance interval (worst case scenario), the variability of the torque would have been estimated equal to 5%. Thus, stochastic approach could be required instead of worst case scenario method for a more precise modeling.

TABLE 2
MEAN VALUE AND STANDARD DEVIATION OF RMS VALUE OF
THE TORQUE

Mode	τ_0	τ_2	τ_4	τ_6
Std (N.m)	560.10^{-5}	$9.22.10^{-5}$	$0.95.10^{-5}$	$2.53.10^{-5}$
Mean value (N.m)	0.4047	0.4047	0.4046	0.4046

V. REFERENCES

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