3D Modeling of Thin Resistive Sheets in the Discontinuous Galerkin Method for Transient Scattering Analysis

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***Abstract*—This papers presents a modeling of thin resistive sheets. An interface condition is used to avoid a fine mesh. The electromagnetic fields are computed with a time domain Discontinuous Galerkin method in order to evaluate the shielding effectiveness. A 3D cavity is treated to illustrate the efficiency of the condition.**

***Index Terms*—Electromagnetic compatibility, Approximation methods, Conducting materials.**

I. Introduction

Many problems in electromagnetic compatibility (EMC) require adequate numerical approaches to evaluate shielding effectiveness. The ability to model features that are small relative to the cell size is often important in electromagnetic simulations. This may lead to subsequent increase in memory and execution time due to a refined mesh around small details of the geometry. Also the quality of the mesh can be strongly affected. The impact of thin sheets is even more consequent for time domain computations.

In order to avoid the spatial discretization of thin sheets different interface conditions have been proposed. In the frequency domain, analytical solutions can be included in a three dimensional model [1]-[2]. In the time domain, an inverse Fourier or Laplace transform is combined to a convolution product. Many papers have been devoted to implement this approach in the FDTD method [3]. Nevertheless the stair casing error present in the FDTD method may affect significantly the numerical results.

The Discontinuous Galerkin (DG) method is a powerful approach for solving time dependent problems [4]. It is based on the local solution of the equations in each cell and uses flux terms to connect adjacent elements. It has the advantage of the unstructured mesh and high spatial order scheme unlike the conventional FDTD. Such a high spatial scheme can reduce the dispersive error induced by the low level of the spatial approximation in the FDTD.

In this paper a specific interface condition is built to replace a thin resistive sheet in EMC 3D models. It allows to take into account conductors with a thickness smaller than the skin depth. This interface condition is implemented in a DG module of GMSH [5].

II. Problem Formulation

Let E, D, H, B and J, be respectively, the electric field, the electric induction, the magnetic field, the magnetic induction and current density. The constitutive laws are:

$$B = \mu H \qquad\qquad D = \epsilon E \tag{1}$$

where ϵ is the permittivity of the medium and μ its permeability. The current density of the conductive medium is such as $J = \sigma E$, with σ the conductivity. The fields satisfy Maxwell's equations in the time domain which constitute a system (2) of 6 unknowns, $E(t, x, y, z)$ and $H(t, x, y, z)$:

$$\begin{cases} \epsilon\partial_t E - \nabla\times H &= -J \\ \mu\partial_t H + \nabla\times E &= 0 \end{cases} \tag{2}$$

The DG variational formulation of (2) in each element T is given by:

$$\begin{cases} \int_T \epsilon\partial_t E\phi - \int_T H\times\nabla\phi - \int_{\partial T}(n\times H)^{num}\phi = -\int_T \sigma E\phi \\ \int_T \mu\partial_t H\psi + \int_T E\times\nabla\psi + \int_{\partial T}(n\times E)^{num}\psi = 0 \end{cases} \tag{3}$$

where ϕ and ψ are test functions.

The interface terms between neighbouring elements are evaluated using numerical fluxes $(n\times E)^{num}$ and $(n\times H)^{num}$ given by:

$$\begin{cases} (n\times H)^{num} &= n\times\frac{\{ZH\}}{\{Z\}} - \alpha(n\times\frac{(n\times[E])}{\{Z\}}) \\ (n\times E)^{num} &= n\times\frac{\{YE\}}{\{Y\}} + \alpha(n\times\frac{(n\times[H])}{\{Y\}}) \end{cases} \tag{4}$$

where $Z = \frac{1}{Y} = \sqrt{\frac{\mu}{\epsilon}}$, $[u] = \frac{u^+ - u^-}{2}$ and $\{u\} = \frac{u^+ + u^-}{2}$. The "+/-" denotes the values in the adjacent element. For $\alpha = 0$, centred fluxes are obtained and the numerical scheme is dispersive [6]. For $\alpha = 1$, upwind fluxes are obtained and the numerical scheme is dissipative [4].

III. The Interface Condition

For a resistive thin sheet of thickness d smaller than the skin depth, a relation between the electromagnetic fields on both sides of the sheet is proposed. This relation given by (5) is built thanks to the analytical two-wire transmission line 1D solution and the continuity of the electric field [7].

$$\begin{cases} E(t,0) = E(t,d) \\ \dfrac{E(t,0)+E(t,d)}{2} = \frac{1}{\sigma d}(H(t,d)-H(t,0)) \end{cases} \tag{5}$$

Let note (E^-, H^-) and (E^+, H^+) the fields on the sides of the sheet and n the outward unit normal. The relation (5) can be reformulated with tangential components of the fields in the general case and becomes:

$$\begin{cases} n\times E^+ = n\times E^- \\ n\times H^+ - n\times H^- = \frac{Y_s}{2}(n\times E^+ + n\times E^-) \end{cases} \tag{6}$$

with $Y_s = \sigma d$, where σ is the conductivity of this sheet.

The resulting flux terms for the interface condition are obtained by taking (4) with $\alpha = 0$ and using (6):

$$\begin{cases} (n\times H)^{num} = n\times \frac{Z^- H^-}{\{Z\}} \\ (n\times E)^{num} = n\times \frac{\{YE\}}{\{Y\}} + n\times \frac{(n\times[H])}{Y_s} \end{cases} \tag{7}$$

IV. Numerical Results

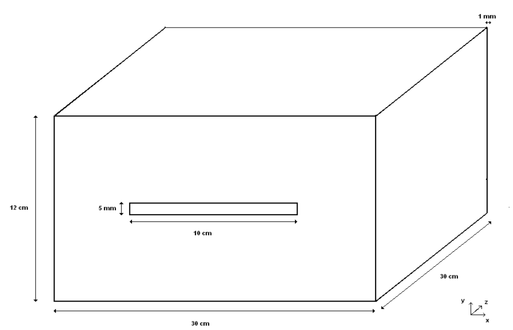

Figure 1: 3D Cavity

A transient scattering problem is studied. Let consider a 3D cavity (Fig. 1), whose dimensions are $a =$ 300 mm, $b =$ 120 mm, $d =$ 300 mm, $l =$ 100 mm, $w =$ 5 mm, $t =$ 1 mm. This cavity is illuminated by an incident Gaussian pulse. A Runge-Kutta scheme is implemented for numerical experiments. The electric field is computed at the center of the cavity. Different cases of conductivity of the sheet are compared: σ = 50 S/m, σ = 10 S/m and Perfect Electric Conductor (PEC).

For the Perfect Electric Conductor (Fig. 2), the electric field is similar to that obtained with the Finite Integration Technique (FIT) method [8]. For the case of finite conductivities (Fig. 3), the field penetrates through the aperture and sides of cavity. The electric field at the center of the cavity remains important for low conductivities and decreases faster with time.

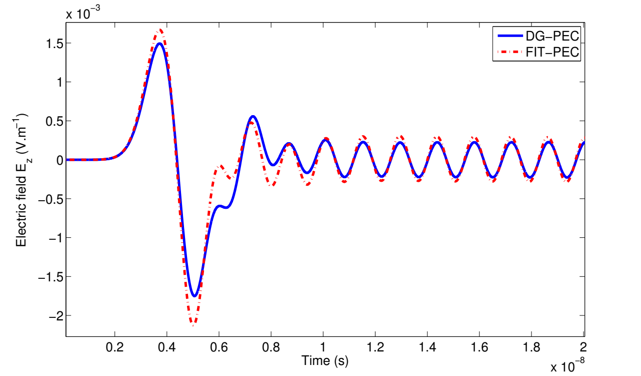

Figure 2: Electric field at the center of the cavity (PEC sheets)

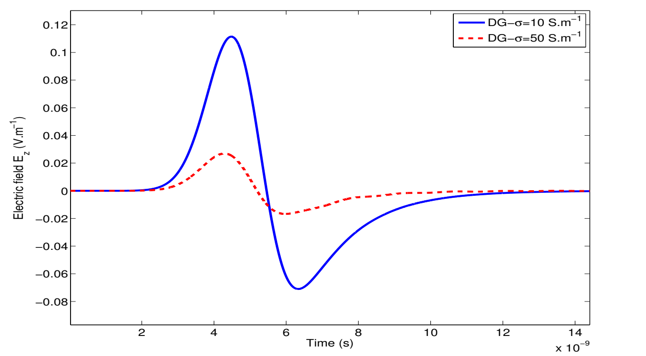

Figure 3: Electric field at the center of the cavity (Conductive sheets)

The wide band response of 3D composite enclosures will be presented at the conference.

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