High Order Finite Elements in T-Ω Method Considering Multiply-Connected Regions

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***Abstract*—This paper describes high order finite elements in the T-Ω method considering multiply-connected regions for the eddy current problems. The inexact Hodge decomposition is instrumental to the discussion. The advantage of the proposed approach is demonstrated by a numerical example.**

***Index Terms*—Eddy currents, Finite element methods.**

I. Introduction

The T-Ω method is one of the effective methods for the finite element analysis of eddy current in the low-frequency electromagnetic applications [1]. In this method, the magnetic field intensity is expressed as the sum of two parts: the gradient of a magnetic scalar potential Ω and electric vector potential $\vec{T}$. The electric vector potential can be interpolated by Whitney edge elements, which use a low-order vector shape function. The T-Ω method has been extended to higher orders using hierarchical scalar and vector elements, assuming that the problem domain is only simply-connected [1]. In this paper, based on the inexact Hodge decomposition, we discuss high order vector elements in the T-Ω method to interpolate the electric vector potential considering multiply-connected regions. Its effectiveness is demonstrated by numerical simulations of TEAM Workshop Benchmark 7.

II. HODGE DECOMPOSITION

Applying the Hodge decomposition to the magnetic field intensity 1-form H gives

$$H = d\phi + \delta A + \chi, \tag{1}$$

where ϕ is a magnetic scalar potential 0-form, and A is a 2-form, and χ represents the harmonic field component, and d is the exterior derivative, and δ is the codifferential operator [2]. The codifferential δ operator can be expressed as [3]

$$\delta = *^{-1} d *, \tag{2}$$

where $*$ is the Hodge star operator. The operators d and δ satisfy

$$dd = \delta\delta = 0 \tag{3}$$

The finite element approximation of (2) gives the matrix formulation

$$[\delta] = [*]^{-1}[d][*], \tag{4}$$

where $[d]$ is the incidence matrix and $[*]$ is the mass matrix. Although $[d]$ and $[*]$ are sparse matrices, $[\delta]$ is generally a full matrix because the inverse of $[*]$ is a full matrix. This implies that δ is not a local operator although d is a local operator. Correspondingly, in the T-Ω method

$$\vec{H} = \nabla\Omega + \vec{T} \tag{5}$$

is not necessary for the basis of the electric vector potential $\vec{T}$ to be divergence-free. Instead, we will use rotational basis functions as introduced in [1][4], which may not be exactly divergence-free, to interpolate $\vec{T}$. Since the rotational basis functions are not necessarily divergence-free in a finite element setting, this decomposition of vector basis into the gradient of a scalar basis and the rotational vector basis is termed the inexact Hodge decomposition [4].

III. BASIS FUNCTIONS

The lowest vector basis function to interpolate $\vec{T}$ is Whitney edge element. As a result, the induced eddy current density, which is the curl of $\vec{T}$, is only a piecewise constant vector in finite element approximation. In order to obtain the more accurate eddy current density distribution, higher order vector basis functions may be needed. In literature there are two types of vector basis: the interpolatory basis and the hierarchical basis [5]. Here, we choose the hierarchical basis since it can handle the harmonic field component naturally. The hierarchical second order vector basis functions are as follows: [1][6]

$$w_{ij}^1 = \varsigma_i \nabla\varsigma_j - \varsigma_j \nabla\varsigma_i, \tag{6}$$

$$w_{ij}^2 = \varsigma_i \nabla\varsigma_j + \varsigma_j \nabla\varsigma_i, \tag{7}$$

$$f_{ijk} = \varsigma_k \varsigma_j \nabla\varsigma_i, \tag{8}$$

where $\varsigma_i, \varsigma_j, \varsigma_k$ are simplex coordinates within a tetrahedron. The basis functions (6) and (7) are defined on the edge connecting vertices i and j, and the basis function (8) is defined on the face connecting vertices i, j and k [6]. The basis function (7) can be expressed as the gradient of the scalar basis function

$$w_{ij}^2 = \nabla(\varsigma_i \varsigma_j) \tag{9}$$

Since (9) is a pure gradient basis function, it can represent the first term of (5). Therefore, the vector basis function (7) can be recycled, and only the rotational basis functions (6) and (8) are needed to interpolate $\vec{T}$.

Generally, the degree of freedom count (DOF) of a finite element approximated system depends on the mesh and orders of the basis functions. However, in multiply-connected regions, DOF of the harmonic field component, which is the number of loops in the computational domain, is independent of the mesh and orders of the basis functions. This suggests that the basis function for the harmonic field component should be the lowest order vector basis function, that is, the Whitney edge element (6). This suggestion was also proposed in [5], but from a slightly different viewpoint. There are several approaches to model the harmonic field component.

Here, the thick-cut as introduced in [7][8] will be applied to model the harmonic field component because from our experience it is very efficient. The thick-cut, which is one layer of tetrahedral elements having their edges passing through cutting surfaces, can be generated by the following two steps [7]:

Step 1. Make surface cuts on the surface of conducting regions. Scan all triangles on the conductor surfaces and for each triangle examine whether each edge forms a loop or not. The triangle with all three edges connected will be added to the set of singly connected surfaces. This process is repeated until there are no more triangles to be added. The rest of the triangles that do not belong to the set of singly connected surfaces create surface cuts on the conductor surfaces.

Step 2. Extend the surface cuts to the non-conducting region. Scan all the tetrahedrons in the non-conducting region. Start from tetrahedrons with a singly connected triangle on the conductor surfaces. If all four triangle faces are singly connected, we add this tetrahedron to the singly connected domain. This process is repeated until no more tetrahedrons are to be added. Finally, the remaining tetrahedrons that are not included in the singly connected domain create the thick-cut.

Scalar basis functions can also be divided into the interpolatory basis and the hierarchical basis [9]. Since the gradient term of (5) is not related to the harmonic field component, one can choose either the interpolatory basis or the hierarchical basis to represent the magnetic scalar potential Ω. Here for simplicity, we choose the second order interpolatory basis to interpolate Ω.

The electric vector potential of impressed source current by external circuits can be interpolated by either the Whitney edge element (the lowest order vector basis) or hierarchical second order vector basis functions. Generally, the accuracy of the Whitney edge element is sufficient to model the source current in real problems [1], so in this paper, we choose Whitney edge elements to interpolate the electric vector potential of impressed source current. The generation of electric vector potential for the impressed source current in multiply-connected regions was detailed in [10].

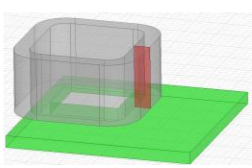
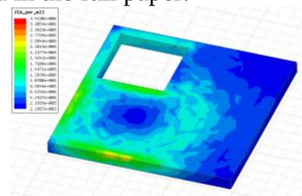


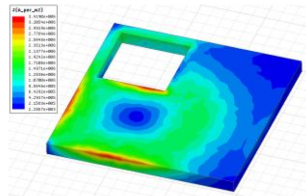
Fig. 1 A thick conductor plate with a hole is placed unsymmetrically in a non-uniform magnetic field. The field is generated by sinusoidal current.

IV. EXAMPLE

The TEAM Workshop Benchmark 7 [11], shown in Fig.1, was solved using the T-Ω method. Since there is a hole in the thick conductor plate, it is a multiply-connected region problem. Fig. 2 presents the induced eddy current densities, which were calculated by applying the first order vector basis functions and the second order vector basis functions, respectively. The results clearly demonstrate that the second order vector basis functions produce more accurate current density than the first order vector basis functions. More results will be presented in the full paper.



(a)



(b)

Fig.2. (a) The induced eddy current calculated by the first order vector basis functions (Whitney elements); (b) The induced eddy current calculated by the second order vector basis functions.

V. CONCLUSION

In this paper, a technique for using high order finite elements in the T-Ω method considering multiply-connected regions for the eddy current problems has been described. In this method, the hierarchical second order vector basis functions are used to interpolate the electric vector potential of induced eddy current; the Whitney edge elements are used to interpolate the harmonic field component; the Whitney edge elements are used to interpolate the electric vector potential of the impressed source currents; the second order interpolatory basis functions are used to interpolate the scalar potential Ω. The numerical results have verified the effectiveness of this high order T-Ω method.

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