Influence of Measurement Errors on Transformer Inrush Currents Using Different Material Models

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Abstract—Inrush currents are an important factor in the design of electrical machines, e.g., transformers. The uncertainties in measurements of magnetic materials parameters propagate via a fitted material law into the simulation results. In this work, the technique of generalized polynomial chaos is used to compute the solution statistics and to perform an analysis of variance, i.e., it is revealed, which measurement points are the most influential. As fitting strategies, we compare the result for the Brauer model and an interproximation approach. This is essential step forward compared to considering only the uncertainty quantification for analytical material curve models.

Index Terms—Electromagnetics, permeability, nonlinearity, measurement uncertainty, transformers, analysis of variance.

I. Introduction

To simulate electromagnetic fields of devices consisting of ferromagnetic materials, one needs the nonlinear material relations, e.g. the reluctivity, which we consider in this work. These material relations are deduced from measurements. The inevitable measurement errors yield uncertainties in the resulting simulations, which have to be considered in order to obtain robust machines.

In this context, the application of Monte-Carlo simulations usually suffers from high computational cost. A more recent developed technique, generalized polynomial chaos [1], [2], is based on an expansion in the space of the random parameters. Thereby, we treat the measurement data as random variables. In the end, so-called Sobol indices [3], [4] allow us to quantify the contribution to the total variance of some output (e.g. device current) with respect to each uncertain measurement.

In this work, we analyze the uncertainties in an interproximation method based on cubic splines which is similarly used in commercial software [5]. This is an important extension of previous works that considered only simple analytical models, e.g. by Brauer [6]. Both models are applied in a finite element simulation of a transformer. We use uniformly distributed random variables to describe measurement errors and compare the resulting inrush currents also by means of Sobol indices for both methods.

II. Uncertainty Quantification

In generalized Polynomial Chaos (gPC) [1], [2], uncertain parameters enter a dynamical system as random vector $Z = (X_1, X_2, \ldots, X_n)$ (on a respective propability space). Depending on the distribution of the Z, one obtains a set of orthogonal polynomials Ψ_k, and thus one can expand of the solution $Y = Y(Z)$ of the system: (up to a certain order M)

$$Y = \sum_{k=0}^{M} y_k \Psi_k(Z) \quad (\text{with} \quad (\mathbb{E}[\Psi_i \Psi_j] = \mathbb{E}[\Psi_i^2]\delta_{ij})). \quad (1)$$

A. Stochastic Collocation

We assume the random vector $Z = (X_1, \ldots, X_n)$ to be composed of independed random variables. Then we can use as basis functions Ψ_k the tensor product of basis functions for each single random variable. We employ stochastic collocation, which is a non-intrusive approach. On a grid in probability space, the dynamical system is evaluated. Discrete projection using Gauss-quadrature rules yields the expansion coefficients y_k of the solution Y.

B. Sobol Indices

To characterize and quantify the influence of each uncertain parameter on the solution (or an output), the so called Sobol indices ($SU_{i_1 \ldots i_k}$) are a very helpful notation [3], [4]. Thereby, the observed variance is split according to all the parameters and to all possible collections of parameters $(i_1, \ldots, i_k)$ (with $i_l \in \{1, \ldots, n\}$). Thus they sum up one. The gPC is used to compute approximations of the Sobol indices.

III. Transformer model

As benchmark we consider a field-circuit coupled system consisting of a 2-D transformer with iron core (no load test). It consists of 600 turns (primary coil), $2.5\,\Omega$, modeled with FEMM software [7], for more details see [8]. It is exited by a sinusoidal voltage source with peak value of $157\,\text{V}$ (chosen s.t. the highest occuring flux density is about $2\,\text{T}$) and $50\,\text{Hz}$. Turning on the voltage source at a zero-crossing, yields an inrush current. The magnitude and the influce of uncertain parameters onto this output current are studied.

A. Curl-Curl Equation

To model electrical transducers, often the curl-curl equation in terms of the magnetic vector potential $\vec{A}$ is used:

$$\sigma \frac{d\vec{A}}{dt} + \nabla \times (\nu \nabla \times \vec{A}) = \vec{J}_s. \quad (2)$$

Here σ, ν and $\vec{J}_s$ denote the conductivity, reluctivity and source current density, respectively. The magnetic field strength $\vec{H}$ and the magnetic flux density $\vec{B} = \nabla \times \vec{A}$ are related by $\vec{H}(\vec{B}) = \nu\vec{B}$ (H-B-curve). It is nonlinear for ferromagnetic materials. Disregarding hysteresis and anisotropy, this gives a scalar law: $H = \nu(B^2)B$ with $H := \|\vec{H}\|_2$ and $B := \|\vec{B}\|_2$.

B. H-B-curve models

Let monotone measurements (B_i, H_i), $i = 0, \dots, N$, with $B_0 = H_0 = 0$ be given. For analysis often analytical models are used, e.g., Brauer's model [6] with parameters k_i:

$$H_{\mathrm{br}}(B) := \nu_{\mathrm{br}}(B^2)B := (k_1 e^{k_2 B^2} + k_3)B. \qquad (3)$$

It is fitted to the measurement data by minimizing $\sum_{i=1}^{N} [(H_{\mathrm{br}}(B_i; (k_1, k_2, k_3)) - H_i)/H_i]^2$. Noisy measurements are handled naturally by this least squares approach.

In commercial software, typically spline interpolation or interproximation [5] are applied. The idea is to construct a cubic spline function f which is monotone and fulfills

$$\int_{H_1}^{H_N} (f''(s))^2 ds \to \min, \quad \sum_{k=1}^{N} ((f(H_k) - B_k)/\omega_k)^2 \leq (c\delta)^2$$

with weights ω_k and tolerance $c\delta$. For simulations using the curl-curl equation (2) the reluctivity is obtained via $\nu_{\mathrm{itprx}}(B^2) := \frac{f^{-1}(B)}{B}$.

C. Uncertainty model for H-B-curve

Due to measurement errors, the reluctivites exhibit uncertainty. This is now model for simulation, by perturbation of certain measurements for the material `M330-35A`. For simplicity and the purpose of demonstration, we perturb four values:

$$\tilde{B}_j = \left(1 + X_j\right)B_j \qquad \text{with} \quad X_j \in U([-\theta, \theta]) \qquad (4)$$

where $\theta = 0.003$. For the interproximation algorithm we use: $c = 5$, $\delta = 0.01$ and $\omega_k = 1$ (for all k).

In the end, the curl-curl equation is solved for both uncertain H-B-curves using gPC. From this Sobol indices are computed.

IV. Results

In this simplified setting small measurement errors were modeled to ensure the monotonicity of the spline. In fact we perturbed the 4 largest field values, $(1.4\,\mathrm{T}, \ 1\,\mathrm{kA/m})$, $(1.47\,\mathrm{T}, \ 1.6\,\mathrm{kA/m})$, $(1.53\,\mathrm{T}, \ 3\,\mathrm{kA/m})$, $(1.76\,\mathrm{T}, \ 12\,\mathrm{kA/m})$, of a set of 12 measurements. These perturbations in the upper part of the H-B-curve yield small variations in the peak of the inrush current in both models: Fig. 1a (Brauer model) and Fig. 1b (interproximation spline). The peak value of the inrush current differs substantially for both models. This could be explained by the fact, that perturbation of the measurements have a more global effect in the Brauer model (this has also influence on the shape of the current).

The Sobol indices SU show a distinct behavior, Fig. 2a (Brauer model) and Fig. 2b (interproximation spline). However, both plots reveal that the largest part of the variance of the current peak is caused by the perturbed measurement with the highest magnitude (corresponding to SU_4). In the Brauer
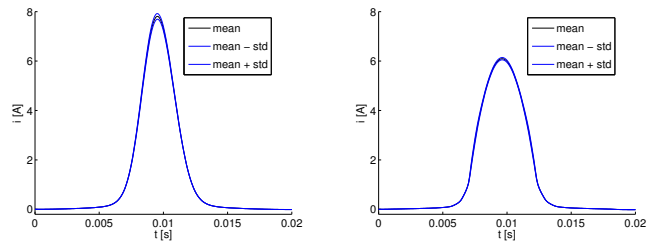
(a) Brauer model. (b) Interproximation spline.

Figure 1: Inrush current (first oscillation) plus/minus standard deviation (4 measurements perturbed at the end).
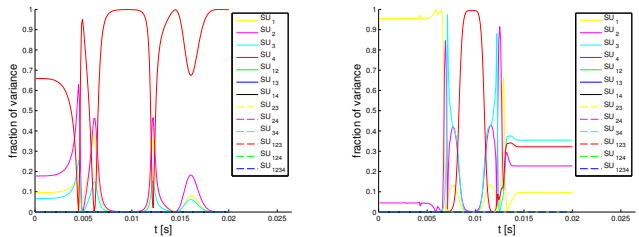
(a) Brauer model. (b) Interproximation spline.

Figure 2: Sobol indices for current (4 measurements perturbed at the end).

model, it exhibits almost globally the largest influence on the inrush current. Whereas in the interproximation case, we find influence of all parameters, depending on the size of current. This fits to the spline nature.

V. Conclusion

We proposed to use spline-based models in the quantification of material uncertainty. The local support of measurement points in a spline interpolation makes the nonlinear simulations much more robust against measurement errors when compared to analytical models. In the full paper we will discuss this for more realistic models and larger deviations in the measurements of the magnetic fields and fluxes.

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