Application of the ACA compression technique for the scattering of Periodic Surfaces

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***Abstract*—The computation of scattering by an infinite periodic structure by an integral equation technique is accelerated by the use of a the ACA method. This compression technique has the advantage to be applied before the building of the matrix. As a result, both assembly and solution phases benefit from the acceleration of computation times. Numerical results assess the efficacy on a problem with a simple periodic surface.**

***Index Terms*—Integral Equations, H-Matrix, ACA**

I. Introduction

Many applications in science and engineering are formulated in terms of scattering by periodic structures. This is especially true in electromagnetics, where periodicity plays an important role in the design of structures. Due to the development of nanotechnology, the importance of periodic boundary value problem is further increased. The development of broadband absorbers, the study of sea surface scattering, the design of uniform antennas arrays, microwave lenses, and artificial dielectric media or photonic cristals are a few examples.

For the analysis of such periodic structures, it usual to solve the problem with numerical method such as Finite Difference Method, Finite Element Method, or Boundary Element Method (BEM). Because of the computation cost of such simulations, it may also be possible to simplify the model by considering as infinite the periodic structures. This is particularly true for BEM for which an appropriate Green function enables to take into account the periodicity. However the *a priori* complexity in $O(N^2)$ restricts BEM to relatively coarse grids. It is then required to propose a method to improve this complexity.

In this work, we consider scattering problems by a perfectly conducting periodic surface Γ in the E-polarization ($u = E_z$) as shown in Fig. 1.

The involved boundary-value problem to solve is then the Helmholtz equation with a Dirichlet condition and a radiation condition at infinity.

II. Integral equation

The scattering problem can be formulated as an integral equation [1] with a single layer potential,

$$\int_\Gamma G(x, x_s) j(x_s) d\gamma(x_s) = -E^{\text{inc}}(x), \ \forall x \text{ on } \Gamma, \tag{1}$$

where E^{inc} is the incident electric field, j the sought density current and G the Green function. In free space, we usually

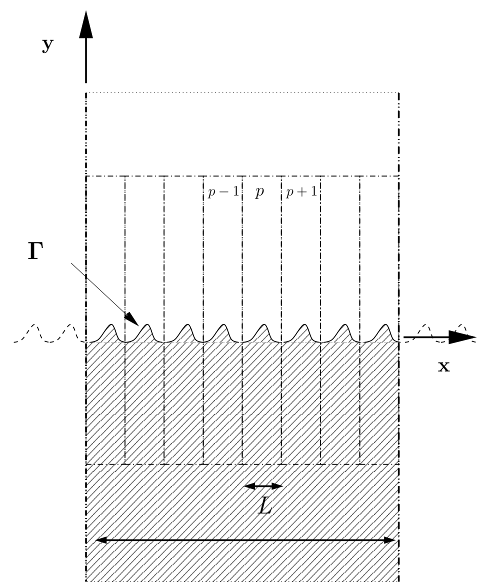

Figure 1: A periodic surface.

consider

$$G(x, x_s) = \frac{1}{4i} H_0^{(2)}(k|x - x_s|) \tag{2}$$

where i is the imaginary unit, k the wave number and $H_0^{(2)}$ the Hankel function of second kind. This definition is replaced in the case of a periodic surface by

$$G(x, x_s) = \sum_{n=-\infty}^{+\infty} \frac{1}{2i\gamma_n L} \ e^{-i\gamma_n|y-y_s|} \ e^{i\alpha_n(x-x_s)} \tag{3}$$

where L is the period of the surface, $\alpha_n = 2\pi n/L + k\theta^{\text{inc}}$ with θ^{inc} the incident angle and $\gamma_n = \sqrt{k^2 - \alpha_n^2}$.

Integral equation (1) is discretized using a Galerkin method with a current density constant per element. It leads *a priori* to a full matrix that should be compressed for memory and computational time efficiency.

III. Matrix compression

A. Hierarchical Matrix

It has been proved that some matrices issued from the discretization of integral equation, as these coming from diffusion problems, can be efficiently represented by a data-sparse format called *hierarchical matrices* often denoted $\mathcal{H}$-matrices [2].

This kind of matrix is constructed on a hierarchical matrix block partition of the original matrix. This partition is related to the geometric positions of the degrees of freedom (dofs) of the discretization; it can be for instance a recursive binary partitioning of this set of dofs. Some blocks of the partition satisfy an admissibility condition and can to compressed. They mainly represent far-field interactions between sets of degrees of freedom. Other non-admissible block has to be fully assembled and they represent near-field interactions.

In order to compress the admissible block, several strategies can be considered as multipole expansion, panel clustering [3]. Here we prefer to focus a purely algebraic approach, the adaptive cross approximation because it is straightforward to implement and multipole expansion for periodic kernel is not yet very efficient [4].

B. Adaptative Cross Approximation

A compression technique (QR algorithm) has been considered for the admissible matrix blocks in a previous work [5], for the same application. Unfortunately this method is limited by the fact that the compression is performed *a posteriori* and consequently it is necessary to assemble the whole matrix. In this work, we perform an Adaptive Cross Approximation (ACA) [6] which can be applied *a priori*. It is an iterative process that computes at each iteration one row and one column (a cross) of the matrix block and an estimate of the error to approximate the block (adaptivity). Thus only selected entries of the matrix block have to be computed.

IV. Numerical Results

We consider here the case of scattering by a sinus surface

$$y = h \sin\left(\frac{2\pi x}{d}\right) \quad (4)$$

where $h = 1$cm and $d = 2$cm and we observe the effect of ACA compression (for a given precision) when the number of degree of freedom increase. The given result is the compression rate (memory storage of the compressed matrix relatively to the full matrix) at various frequencies (1Ghz to 1Thz).

This result shows that the method works with the expected asymptotic behavior. As a result, the $\mathcal{H}$-Matrices are known to expect (whatever is the compression technique) an increase of the memory storage in $N\log(N)$, which is obtained here. In terms of computation time, the improvement is not fully observed because of an important cost of the assembly of diagonal uncompressed blocks, see Fig. 3.

Nevertheless a gain of factor 2 to 5 is obtained for 12800 unknowns (comparatively to compression rate 1% to 3%). At higher size, direct assembly is possible to conjecture improved gains. This work is yet under studie, a specific improvement of the diagonal assembly being necessary to obtain all the performances of the method.

Note : for this application, the solution of linear system by an iterative solver is fast comparative to the assembly. It has then not be studied in details yet. It will be useful to improve it when the size of the system will increased and the diagonal assembly will be fully improve.

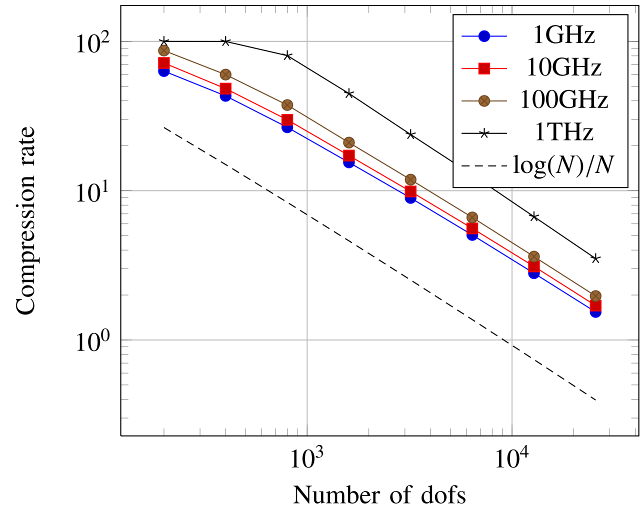

Figure 2: Compression rate vs the number of degrees of freedom (dofs).

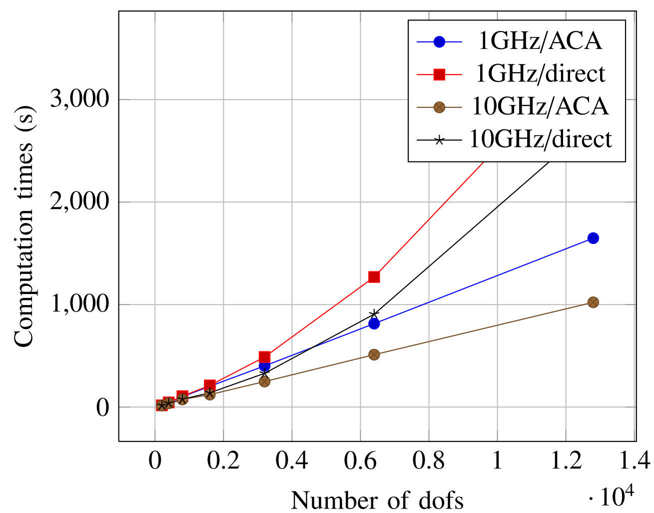

Figure 3: Computation times.

References

[1] K. Zaki and A. Neureuther, "Scattering from a perfectly conducting surface with a sinusoidal height profile: TE polarization," *Antennas and Propagation, IEEE Transactions on*, vol. 19, no. 2, pp. 208 – 214, 1971.

[2] W. Hackbusch, "A sparse matrix arithmetic based on h-matrices. part i: Introduction to h-matrices," *Computing*, vol. 62, pp. 89–108, 1999. [Online]. Available: http://dx.doi.org/10.1007/s006070050015

[3] W. Hackbusch and Z. Nowak, "On the fast matrix multiplication in the boundary element method by panel clustering," *Numerische Mathematik*, vol. 54, pp. 463–491, 1989. [Online]. Available: http://dx.doi.org/10.1007/BF01396324

[4] N. N. Y. Otani, "A periodic fmm for maxwell's equation in 3d and its application to problems related to photonic crystals," *Journal of Computational Physics*, vol. 227, no. 9, pp. 4630–4652, Feb. 2008.

[5] S. Tournier, J. Girardin, J.-R. Poirier, and P. Borderies, "Analysis of QR-compression techniques for improving electromagnetic scattering computation by periodic rough surfaces." in *AMPERE, 13th International Conference on Microwave and RF Heating*, september 2011.

[6] M. Bebendorf, "Approximation of boundary element matrices," *Numerische Mathematik*, vol. 86, no. 4, pp. 565–589, Oct. 2000. [Online]. Available: http://www.springerlink.com/index/10.1007/PL00005410