Flexible BiCGStab to solve the Discretized EFIE in Scattering Computations

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***Abstract*—The Bi-Conjugate Gradient Stabilized method is able to handle and solve large linear systems, such as those arising from electromagnetic scattering inverse problems formulated by EFIE and discretized by the Galerkin Method of Moments by means of well-known Rao-Wilton-Glisson method. Nevertheless, preconditioning phase should be taken into account very carefully, in order to maintain the advantages of the use of Krylov subspaces in solving sparse inverse linear problems. In this sense, flexible version of the Bi-Conjugate Gradient Stabilized algorithm received small attention, but could be really worthwhile for a correct and fast preconditioning. In this work, we propose a numerical study on its performances. Results demonstrate that Flexible Bi-Conjugate Gradient Stabilized is faster and more robust than the standard version coupled with diagonal or ILU preconditioning, at least in specific cases involving electromagnetic inverse scattering.**

***Index Terms*—Integral equations, Iterative methods, Moment methods.**

I. Introduction

Computational Electromagnetics (CEM) is a well known discipline related to modelling electromagnetic phenomena through computational discretization of analytic formulation. Here, a number of integral formulations can be exploited in order to model electromagnetic scattering and radiation. The most extensively exploited method is maybe the standard form of the Electric Field Integral Equation (EFIE), even in spite of some well-known limitations (see [1] and references therein). When a real-world electromagnetic problem is approached, the discretization arising from the application of EFIE provides a large, dense and non-Hermitian linear system of equations. If preconditioned by a suitable technique [2], the problem could be satisfactorily solved by exploiting the Krylov Subspaces (KS). KS method (KSM), in fact, is a very fruitful iterative technique for solving large linear systems involving sparse matrices in an efficient and fast way. It avoids matrix-matrix operations. During the years, the joint use of a preconditioner with KSM has been widely inspected. Nowadays, the Generalized Minimal Residual Method (GMRES) combined with Incomplete LU (ILU) factorization is worldwidely accepted as a good compromise (please, refer to [2] and references within). Even if GMRES satisfies the requirement of minimal residual solution over the associated Krylov subspaces, it needs computing memory that grows linearly with the number of iterations and could become rapidly prohibitive [2]. A restarted version of GMRES [3], the so called GMRES(m), can overcome this limitation. The restart parameter m is very useful to optimize convergence rate as well as memory requirements, but the assignment of a starting a-priori value is a very difficult task. In case of large and dense non-Hermitian linear systems, huge amount of memory could be necessary in order to avoid out-of-memory errors during computer simulations. But, if it would be possible to fix a predefined amount of memory for each iteration, then the procedure could success: this is the case of the Bi-Conjugate Stabilized (BiCGStab) method. It is less popular than GMRES, but it has been successfully exploited in CEM to deal with large EM problems. Its performance can be improved providing that the preconditioner is based on Krylov suspaces itself [4], so building the so called Flexible BiCGStab method. A representative scenario is that the preconditioning requires a linear solve with a second iterative method, in which case "inner iterations" are used to mean preconditioning and "outer iterations" are used to mean the flexible Krylov method itself [5]. Flexible BiCGStab has been extensively exploited in various engineering fields (see for example [6] and references therein). But, at the best of our knowledge, CEM has been small focused on this framework. In this work, we aim to propose Flexible BiCGStab on CEM for solving the dense non-Hermitian linear systems arising from discretization of EFIE, presenting results of a numerical study on its performances.

II. Basics on Flexible BiCGStab

The mathematical ideas and principles characterizing Flexible BiCGStab (see [3] and references within) are here proposed and discussed. Basically, Flexible BiCGStab searches for an approximate solution of the matrix system (i.e. the discretized EFIE)

$$\mathbf{Z}\mathbf{i} = \mathbf{v} \tag{1}$$

of the form

$$\mathbf{i}_n = \mathbf{i}_0 + \mathcal{K}_n(\mathbf{r}_0, \mathbf{Z}) \tag{2}$$

where $\mathbf{i}_0$ is any initial guess for the solution of Eq.1, $\mathbf{r}_0$ is the residual vector, and $\mathcal{K}_n(\mathbf{r}_0, \mathbf{Z})$, defined as

$$\mathcal{K}_n(\mathbf{r}_0, \mathbf{Z}) = span\{\mathbf{r}_0, \mathbf{Z}\mathbf{r}_0, \ldots, \mathbf{Z}^{n-1}\mathbf{r}\} \tag{3}$$

Table I
SETTINGS AND RESULTS OF NUMERICAL EXPERIMENTS

Method	Precond.	Conv.	n. Iterat.	Residual
BiCGStab	none	no	500	$\sim 2.78 \cdot 10^{-5}$
Flexible BiCGStab	**X**	yes	10	$\sim 2.96 \cdot 10^{-8}$
BiCGStab	ILU	out of memory	-	-

is the n-th Krylov subspace generated by the couple $(\mathbf{r}_0, \mathbf{Z})$. It is possible to demonstrate that any vector in $\mathcal{K}_n(\mathbf{r}_0, \mathbf{Z})$ can be written as

$$\mathbf{i}_n = \mathbf{i}_0 + q_{n-1}(\mathbf{Z})\mathbf{r}_0 \tag{4}$$

where q_{n-1} is a suitable polynomial of a degree at most $n-1$. This implies that the residual $\mathbf{r}_n$ is associated to a so called residual polynomial p_n which degree is n at most. The approximate solution $\mathbf{i}_n$, or equivalently the associate residual polynomial p_n, is found by Flexible BiCGStab if the residual $\mathbf{r}_n$ fulfils the following orthogonality condition

$$\mathbf{r}_n \perp \mathcal{K}_n(\hat{\mathbf{r}}_0, \mathbf{Z}^H) \tag{5}$$

where $\hat{\mathbf{r}}_0$ is an arbitrary vector satisfying the condition

$$\langle \hat{\mathbf{r}}_0, \mathbf{r}_0 \rangle = 1 \tag{6}$$

More precisely Flexible BiCGStab tries to construct a sequence of vectors $\{\mathbf{i}_n\}$ such that $\mathbf{i}_n \rightarrow \mathbf{i}$ for $n = 1, 2, 3, \ldots$ in a given tolerance ε. The i-th iteration consist of two levels: an outer level necessary to compute $\mathbf{i}_n$ starting from $\mathbf{i}_{n-1}$, and an inner sub-optimal preconditioner provided by a secondary KSM. In this way, the asimptotic complexity $O(n^2)$ of the overall procedure (at the step n) is unvaried. A pseudocode of the Flexible BiCGStab method is reported below.

Algorithm 1 Right Preconditioned Flexible BiCGStab Algorithm

Compute $\mathbf{r}_0 = \mathbf{v} - \mathbf{Z}\mathbf{i}_0$
Choose $\hat{\mathbf{r}}_0$ s.t. $\langle \mathbf{r}_0, \hat{\mathbf{r}}_0 \rangle \neq 0$
Set $\beta = \|\mathbf{r}_0\|$
for $i = 1, 2, \ldots$ **do**
  Set $\rho_{i-1} = \langle \hat{\mathbf{r}}, \mathbf{r}_i \rangle$
  if $\rho_{i-1} = 0$ **then**
    Method Fails
  end if
  if $i = 1$ **then**
    $\mathbf{p}_i = \mathbf{r}_i$
  else
    $\beta_{i-1} = \frac{\rho_{i-1}}{\rho_{i-2}} \frac{\alpha_{i-1}}{\omega_{i-1}}$
    $\mathbf{p}_i = \mathbf{r}_{i-1} + \beta_{i-1}(\mathbf{p}_{i-1} - \omega_{i-1}\mathbf{v}_{i-1})$
  end if
  Solve $\mathbf{P}\hat{\mathbf{p}} = \mathbf{p}_i$ by a secondary KSM
  $\mathbf{v}_i = \mathbf{Z}\hat{\mathbf{p}}$
  $\alpha_i = \frac{\rho_{i-1}}{\langle \hat{\mathbf{r}}, \mathbf{v}_i \rangle}$
  $\mathbf{s} = \mathbf{r}_{i-1} + \alpha_i \mathbf{v}_i$
  if $\|\mathbf{s}\|$ is small enough **then**
    Set $\mathbf{i}_i = \mathbf{i}_{i-1} + \alpha_i \hat{\mathbf{p}}$ and Stop
    Solve $\mathbf{P}\hat{\mathbf{s}} = \mathbf{s}$ by a secondary KSM
    $\mathbf{t} = \mathbf{Z}\mathbf{s}$
    $\omega_i = \frac{\langle \mathbf{t}, \mathbf{s} \rangle}{\langle \mathbf{t}, \mathbf{t} \rangle}$
    $\mathbf{i}_i = \mathbf{i}_{i-1} + \alpha_i \hat{\mathbf{p}} + \omega_i \hat{\mathbf{s}}$
    $\mathbf{r}_i = \mathbf{s} - \omega_i \mathbf{t}$
    Check convergence, continue if necessary for continuation it is necessary that $\omega_i \neq 0$
  end if
end for

III. NUMERICAL RESULTS

As a test case we considered the plane wave scattering by a perfect conducting metallic sphere with radius $r = 0.5$ m at the frequency $f = 150$ MHz. A computer code implementation of the Rao-Wilton-Glisson Method of Moments and of the Flexible BiCGStab method has been developed exploiting the MATLAB environment. Simulations have been carried out using a PC with an Intel Core^{TM} 2 Duo processor at 1.66 GHz and 4 GB of main memory. The conducting sphere has been discretized with $n = 14850$ edges. All the results have been obtained fixing an accuracy $\varepsilon = 10-6$ and a maximum number of iterations $maxit = 500$ for the outer level, while fixing $\varepsilon = 10^{-1}$ and $maxit = 100$ for the inner level. The initial guess vector is set equal to the null vector for both levels. Different KSMs have been considered for the inner level but in what follows only results considering the BiCGStab as inner method are shown. Table I compares the performances of the Flexible BiCGStab, in which the preconditioning step of Equation 6 is carried out exploiting the antihermitian component of Z, with those provided by i) the standard BiCGStab algorithm and ii) its some preconditioned implementations. Numerical results evidence how the Flexible BiCGStab reduces significantly both the overall number of iterations and the convergence elapsed time.

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