Force Calculation from the Finite Element Solution Avoiding Nonzero Local Forces in the Air Region

Takeshi Mifune
Kyoto University
Kyoto-Daigaku-Katsura, Nishigyo-Ku, Kyoto, 6158510, Japan
mifune@fem.kuee.kyoto-u.ac.jp

***Abstract*—This paper describes method to calculate magnetic forces based on finite element analyses. Local forces obtained by conventional methods frequently appear in the interior of linear, isotropic and homogeneous regions, e.g., an air region. The present method gives local forces that exactly vanish in such regions where no force should appear.**

***Index Terms*—Electromagnetic Forces, Finite Element Methods, Nodal Force Method, Edge-Element.**

I. Introduction

Various methods have been used to calculate forces acting on magnetized bodies from finite element solutions [1]–[7]. Different from the equivalent charge or current approach, the methods based on the virtual work principle [1]–[3] or a local application of Maxwell stress tensor [4], [5] give local force distributions in the form of nodal forces, although there have been some arguments as to the meaning of these local forces.

Because finite element solutions do not exactly satisfy both Faraday's and Ampere's laws, the local forces calculated by those methods—except for those using two dual formulations [3]—can be nonzero at the nodes in the interior of a linear, isotropic and homogeneous region, for instance, an air region. In this study, we discuss a method to calculate forces that guarantees that the local forces vanish exactly in those regions, where theoretically the force should be zero.

II. Nodal Force Calculation from Finite Element Analysis

The nodal force $\boldsymbol{f}_i$ associated with a node i is given by

$$\boldsymbol{f}_i = \int w_i \boldsymbol{f} \, dV , \tag{1}$$

where $\boldsymbol{f}$ is the force density and w_i is the weight function associated with the node. If the domain considered is composed of element-wise linear, isotropic and homogeneous media, the force density is concentrated on the surface of each element (Fig. 1).

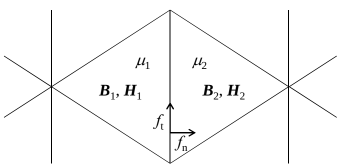


Fig. 1. Surface Force Density on a Boundary between Different Materials

A direct application of the Maxwell stress tensor to finite element solutions [4], [5] leads to the following normal and tangential components of the surface force density:

$$f_{\mathrm{n}} = \frac{1}{2}\Delta\left(B_{\mathrm{n}} H_{\mathrm{n}}\right) - \frac{1}{2}\Delta\left(B_{\mathrm{t}} H_{\mathrm{t}}\right), \tag{2}$$

$$f_{\mathrm{t}} = \Delta\left(B_{\mathrm{n}} H_{\mathrm{t}}\right). \tag{3}$$

Here, $\boldsymbol{B}$ and $\boldsymbol{H}$ are the magnetic flux density and magnetic field intensity respectively, and subscripts n and t represent the respective normal and tangential components, and $\Delta(B_{\mathrm{n}} H_{\mathrm{n}}) = B_{2\mathrm{n}} H_{2\mathrm{n}} - B_{1\mathrm{n}} H_{1\mathrm{n}}$, $\Delta(B_{\mathrm{t}} H_{\mathrm{t}}) = B_{2\mathrm{t}} H_{2\mathrm{t}} - B_{1\mathrm{t}} H_{1\mathrm{t}}$, and $\Delta(B_{\mathrm{n}} H_{\mathrm{t}}) = B_{2\mathrm{n}} H_{2\mathrm{t}} - B_{1\mathrm{n}} H_{1\mathrm{t}}$. It should be noted that f_{t} does not necessary vanish in a region with no source current, because in general the finite element solution does not satisfy both continuities of the normal component of $\boldsymbol{B}$ and tangential component of $\boldsymbol{H}$. For the same reason, f_{n} and f_{t} can be nonzero even in the interior of an air region.

In the magnetic vector potential formulation, the essential problem is that the finite element solution directly gives a good estimate for $\Delta(B_{\mathrm{n}} H_{\mathrm{n}})$ in (2) but not for $\Delta(B_{\mathrm{t}} H_{\mathrm{t}})$ or $\Delta(B_{\mathrm{n}} H_{\mathrm{t}})$. The following section presents a new approach to avoid the emergence of the undesirable forces, by evaluating $\Delta(B_{\mathrm{t}} H_{\mathrm{t}})$ and $\Delta(B_{\mathrm{n}} H_{\mathrm{t}})$ via the magnetizing currents.

III. Evaluation via the Magnetizing Currents

For simplicity, we consider the magnetostatic problem:

$$\nabla \times \left(\mu^{-1} \nabla \times \boldsymbol{A}\right) = \boldsymbol{J}_0 , \tag{4}$$

where $\boldsymbol{A}$, $\boldsymbol{J}_0$, and μ denote the magnetic vector potential, the source current density, and the magnetic permeability, respectively. Let

$$K\boldsymbol{x} = \boldsymbol{b} \tag{5}$$

be the matrix equation arising from the finite element formulation. Here, with the shape functions $\boldsymbol{w}_i$,

$$K_{ij} = \int \mu^{-1} \nabla \times \boldsymbol{w}_i \cdot \nabla \times \boldsymbol{w}_j dV , \tag{6}$$

$$b_i = \int \boldsymbol{w}_i \cdot \boldsymbol{J}_0 dV . \tag{7}$$

If we simply calculate the magnetizing current density $\boldsymbol{J}_{\mathrm{m}}$ from the rotation of $\boldsymbol{B}$ obtained by the finite element solution, $\boldsymbol{J}_{\mathrm{m}}$ can be nonzero on all the faces of the finite element mesh. For the purpose of this work, we consider instead an equivalent equation with the vacuum permeability μ_0:

$$\nabla \times \left(\mu_0^{-1} \nabla \times \boldsymbol{A}\right) = \boldsymbol{J}_0 + \boldsymbol{J}_{\mathrm{m}} , \tag{8}$$

and the corresponding matrix equation:

$$K^0 \boldsymbol{x} = \boldsymbol{b} + \boldsymbol{b}^{\mathrm{m}} , \tag{9}$$

$$K_{ij}^0 = \int \mu_0^{-1} \nabla \times \boldsymbol{w}_i \cdot \nabla \times \boldsymbol{w}_j dV \ , \tag{10}$$

$$b_i^m = \int \boldsymbol{w}_i \cdot \boldsymbol{J}_{\mathrm{m}} dV \ . \tag{11}$$

Here $\boldsymbol{b}^{\mathrm{m}}$ is the vector associated with $\boldsymbol{J}_{\mathrm{m}}$, and from (5) and (9) it should be given by

$$\boldsymbol{b}^{\mathrm{m}} = \left(K^0 - K\right)\boldsymbol{x} \ . \tag{12}$$

Note that, if $b_k = 0$ and all elements surrounding edge k consist of a same medium, $b_k^{\mathrm{m}} = 0$ is guaranteed. In a region with no source current, the magnetizing currents considered therefore appear only on the boundary between different media. The force density calculated from $\nabla \times \mu \boldsymbol{H} = \mu_0\left(\boldsymbol{J}_0 + \boldsymbol{J}_{\mathrm{m}}\right)$ has only a normal component on the faces of the elements, and it vanishes except interfaces between different media.

IV. Numerical Test

Fig. 2 shows a 2D test model, which is discretized by a triangular mesh. Nodal forces acting on the magnetic material were calculated by the conventional and proposed nodal force (NF) methods. Although it is appropriate to calculate $\boldsymbol{J}_{\mathrm{m}}$ so as to satisfy (11) exactly, we evenly distributed each b_i^{m} to the neighbor surfaces, regarding b_i^{m} as the line current flowing along the edge (node in 2D) i.

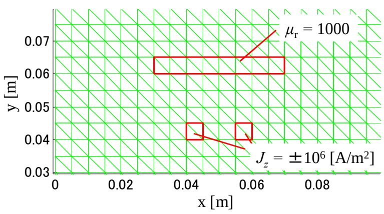
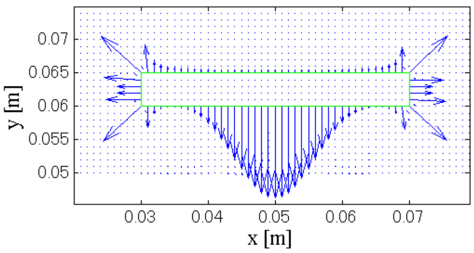
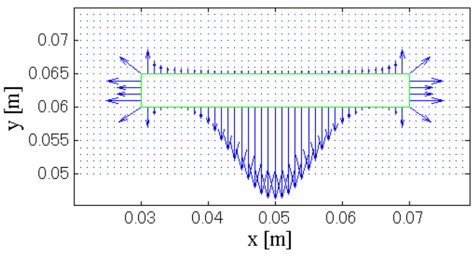


Fig. 2. Test Model and 20×20 Mesh



(a). Conventional NF Method

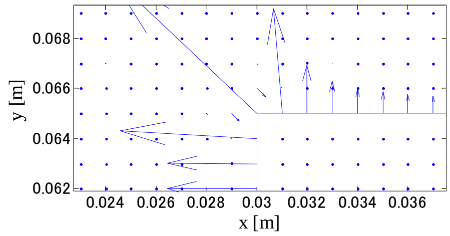


(b) Proposed NF Method

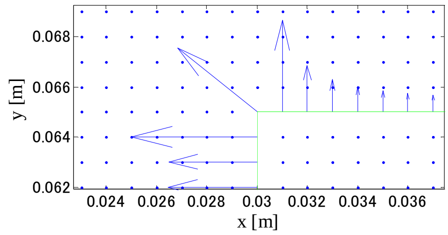
Fig. 3. Nodal Forces (100×100 Mesh)

The two methods give almost the same results (Fig. 3), except for around the corners of the magnetized material (Fig. 4). With the proposed method, forces acting in the air region and artificial tangential components on the surfaces clearly do not appear. Figure 5 shows the total forces acting on the body for different mesh sizes. Using a sufficiently fine discretization, the forces computed by the conventional and proposed methods converge to the same value.

More details of the present method and related results will be reported in the full paper.



(a). Conventional NF method



(b) Proposed NF Method

Fig. 4. Nodal Forces (Enlarged Views)

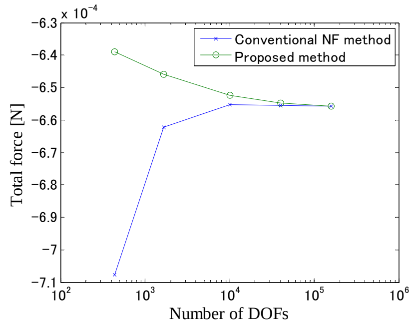


Fig. 5. Total Forces

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