Robust Global Optimization of Electromagnetic Designs Utilizing Gradient Indices and Kriging

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Abstract—Since uncertainties in design variables are inevitable an optimal solution must consider the robustness of the design. A methodology based on the use of first-order and second-order gradient indices is proposed introducing the notion of gradient sensitivity. A kriging method assisted by algorithms exploring the concept of rewards is utilized to facilitate function predictions for the robust optimization process. The performance of the proposed algorithms is assessed through a series of numerical experiments and the TEAM Workshop Problem 22.

Key words—Design optimization, pareto optimization.

I. INTRODUCTION

The concept of the Gradient Index (GI) was introduced and explored in [1]-[3]. The method transforms a problem into a multi-objective optimization by simultaneously minimizing the function itself and its gradient index, thus creating pareto fronts. This looks a very promising approach but has a certain weakness that it is difficult to select the preferred solution depending on the different size of the uncertainty. Moreover, to fully benefit from the method the sensitivity computations must be incorporated into the finite element code, something which is not possible when commercial software is used.

In this paper we pursue similar ideas but introduce some important changes. First, we propose to use the concept of gradient index sensitivity, explained below. Secondly we investigate how the second order gradient index could assist in the process. Finally, instead of calculating the objective function using the computationally expensive finite element program, we use a kriging prediction. In other words, the objective function uses the kriging method [4] assisted by algorithms that balance exploration and exploitation ([5], [6]) using the concept of rewards [7]. This optimization strategy has been shown previously to be very efficient and has the advantage that it can link with any finite element software.

II. ROBUST OPTIMISATION

In conventional optimization the minimum (maximum) of an objective function is sought while the searching space is limited through a set of constrains. Once the global optimum has been found the problem is considered to have been solved. When practical devices are designed, however, we need to recognise that almost all parameters (design variables) are subject to uncertainties (manufacturing tolerances, variation of material properties, etc) and thus not just the value but also the shape of the optimum needs to be considered in the neighbourhood of the selected design; this is demonstrated by the examples of Figs. 1 and 2. A theoretical optimum may therefore be abandoned in favour of a 'worse' but more robust design; however, the decision will depend on the size of the uncertainties involved. For this reason having a pareto front instead of a single solution may be preferable.

A. Multi-Objective Robust Optimization using Gradient Index

Consider a one-variable test function [1]-[3] (see Fig. 1)

$$f(x) = 3 - \frac{3.5}{1+(x-5)^2} - \frac{2.2}{1+(x-15)^2/10} - \frac{1.2}{1+(x-25)^2/30}. \quad (1)$$
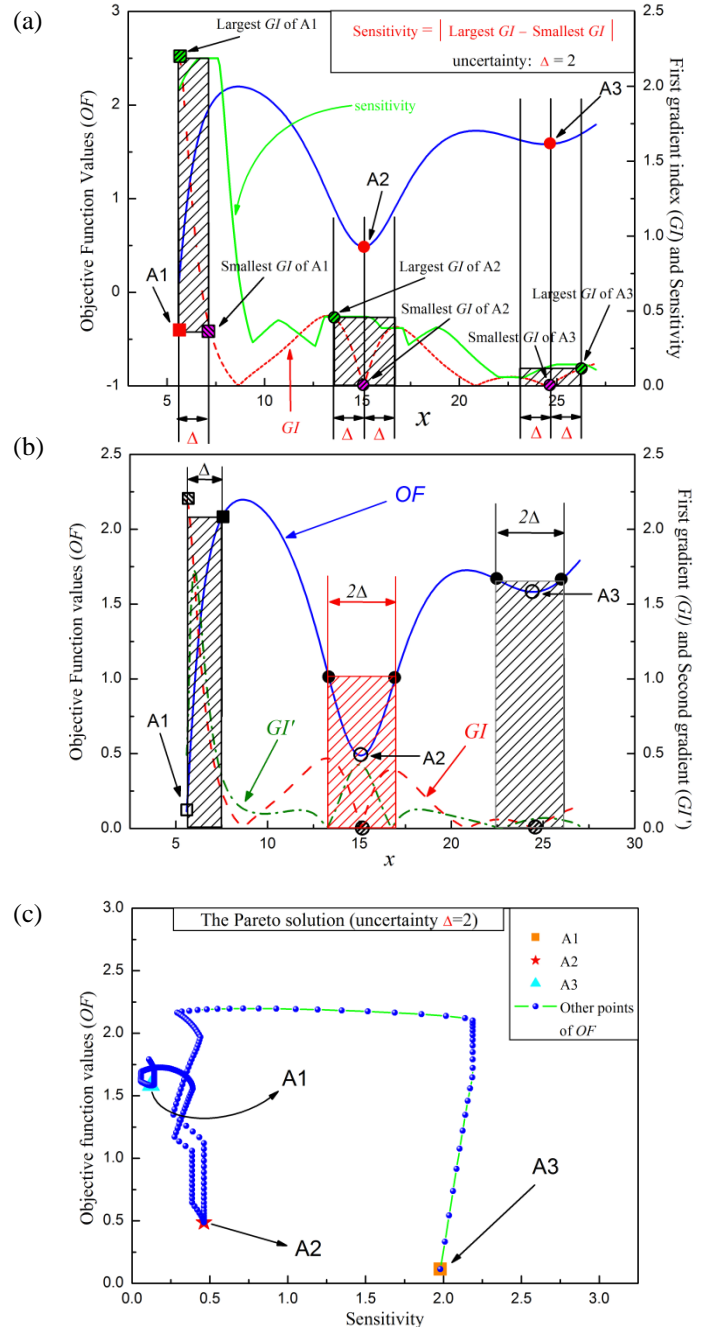

Fig. 1. Example of a robust design for a one-variable problem
(a) Objective function, the gradient index and sensitivity,
(b) First and second order gradients, (c) Pareto solution.

The uncertainties may be either specified directly (e.g. as machining tolerances, say Δ) or defined mathematically as

$$U(x) = \{\xi \in R^n | x - k\sigma \le \xi \le x + k\sigma\} \quad (2)$$

where σ is standard deviation of uncertain variables and k is determined by a confidence level [2].

One way of incorporating robustness into the mainstream optimisation process is by adding the gradient index [1] as a second objective and formulating the problem as

$$\begin{array}{lll} \text{Minimize} & f(x) & x \in R^n \ (x_L \le x \le x_U) \\ \text{Minimize} & GI(x) = \max_{1 \le i \le n} |\partial f(x)/\partial x_i| & (3) \\ \text{Subject to} & g_i(x) \le 0, i = 1, \dots, m \end{array}$$

As shown in Fig. 1(a) point A is the theoretical global optimum. However, any small change in the variable x results in a large variation of the objective function; thus A is not a robust design and points B or C might be preferred. The final decision, however, is not straightforward and is influenced by the size of the uncertainty. We define a sensitivity of the gradient as the difference between the largest and the smallest value of the GI within the uncertainty range; as shown in Fig. 1(a) the shape of this sensitivity carries useful information. Finally, a second order gradient may also be useful (Fig. 1(b)).

The second example of Fig. 2 considers a 'sharp' global minimum, 'shallow' local minimum and a 'plateau'.
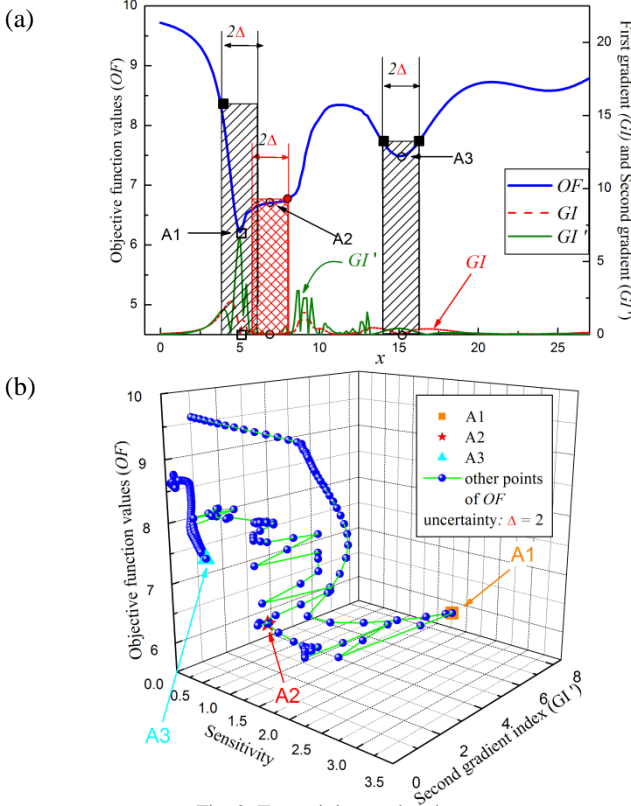
(a)

(b)

Fig. 2. Two minima and a plateau
(a) Objective function and the gradient indices, (b) Pareto solution.

B. TEAM 22 Problem

The final case refers to TEAM Benchmark Problem 22, the optimization of a superconducting magnetic energy storage system (SMES) [8]. Table I compares one typical result from literature with our AWEI algorithm (kriging with Adaptive Weighted Expected Improvement) [5], [6], while Fig. 3 demonstrates the convergence process of AWEI.

TABLE I
PERFORMANCE COMPARISON OF ALGORITHMS

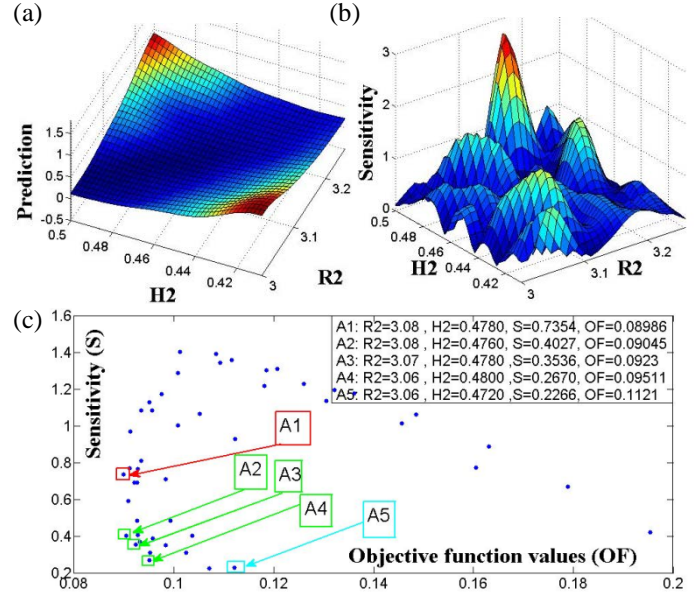| Algorithm	R2(m)	h2/2(m)	Best OF	Iterations
RBF	3.06	0.236	0.088	240
AWEI (Kriging)	3.08	0.239	0.089	38

Radial basis functions (RBF) [9]; AWEI (Kriging) [6].

Fig. 3. TEAM Workshop Problem 22 (a) Prediction by kriging with AWEI
$(2.6 \le R_2 \le 3.4, 0.408 \le H_2 \le 2.2, D_2=0.394,$ other parameters fixed),
(b) Sensitivity with respect to R_2 and H_2, (c) Pareto solution.

Finally, it is also possible to define a 'function uncertainty' as the second objective or use the notion of 'curvature'. Lack of space does not allow elaborating on those issues so further discussion will be deferred until the full version of the paper.

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