Non Linear Multiphysics Analysis and Multiobjective Optimization in Electro-Heating Applications

P. Di Barba[1,2], I. Dolezel[3], P. Karban[4], M.E. Mognaschi[1], A. Savini[1]

[1]University of Pavia, Dept of Industrial and Information Engineering
Via Ferrata 1, I-27100 Pavia, Italy
eve.mognaschi@unipv.it, savini@unipv.it

[2]University of Padova, Dept of Industrial Engineering
Via Gradenigo 6/A, I-35031 Padova, Italy
paolo.dibarba@unipv.it

[3]Academy of Sciences of the Czech Republic, Inst. of Thermomechanics, v.v.i. Dolejškova 1402/5, 182 00 Praha 8, Czech Republic

[4] Faculty of Electrical Engineering, University of West Bohemia, Univerzitní 8, 306 14 Plzen
karban@kte.zcu.cz

***Abstract*— The design optimization of an induction heating device is considered. The non-linear multiphysics analysis is carried out by means of finite-element method, while the optimal design problem is solved with NSGA-II genetic algorithm. A comparison with the results obtained by a simplified linear analysis is shown. The original contribution of the paper is the Pareto front identification for a design problem in which the field analysis is multi-physics, dynamic and non-linear.**

***Index Terms*—Coupled multi-physics problems, non-linear equations, finite element method, optimization and design.**

I. INTRODUCTION

Multiphysics and multiobjective optimization problems currently stand at the frontier of research in inverse electromagnetics. The complexity of the analysis of coupled problems, in fact, and that of the connected solution of multiobjective optimization make the overall design problem almost prohibitive, particularly in the case of non-linear analysis. In particular, in industrial design problems, modelling devices whose operation is based on an interaction of several physical fields requires to consider their mutual influences, both for analysis and synthesis. A widely spread technique of material processing is induction heating of metal bodies, mostly used for improving their mechanical properties, with the purpose to obtain a prescribed temperature profile [1]; inverse induction-heating problems are investigated also in different areas, like *e.g.* clinical hyperthermia [2] and [3]. With reference to an engineering application in electro-heating, in a previous paper [4] the multi-objective optimization problem was solved by considering linear assumptions for the multiphysiscs analysis. The paper shows a comparison of the previous results with those obtained by a non-linear analysis of the device under investigation.

II. MULTIPHYSICS NON-LINEAR FORWARD PROBLEM

The induction heating of a steel cylinder placed in a crucible furnace (Fig.1) is investigated as the case study. The inductor carries a harmonic current I of density $\boldsymbol{J}_{\mathrm{ext}}$ and angular frequency ω that induces a current density $\boldsymbol{J}_{\mathrm{ind}}$ in the charge and, therefore, heat due to generated Joule losses. The current density induced in the inductor is neglected.

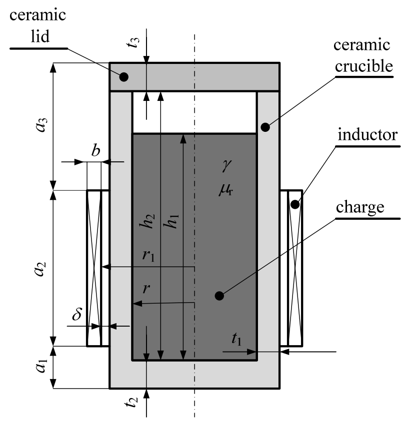

Fig. 1. Induction heating device

The arrangement is considered to be axisymmetric. The relative permeability $\mu = \mu_0\mu_r$ of the charge (μ_r = 500) is assumed to be independent of both magnetic flux density and temperature. The mathematical model, which includes the magnetic equation in time-harmonic conditions and the thermal equation in transient time domain, reads

$$\nabla \times \left(\nabla \times \underline{\boldsymbol{A}}\right) + \mathrm{j}\mu\omega\gamma(T)\,\underline{\boldsymbol{A}} = \mu \underline{\boldsymbol{J}}_{\mathrm{ext}}\,, \tag{1}$$

$$\nabla \cdot \left(\lambda(T)\nabla T\right) = \frac{\partial\left(\rho(T)c_{\mathrm{p}}(T)T\right)}{\partial t} - p_{\mathrm{J}} \tag{2}$$

with $p_{\mathrm{J}} = \left[\gamma(T)\right]^{-1}\left|\underline{\boldsymbol{J}}_{\mathrm{ind}}\right|^2$ and $\underline{\boldsymbol{J}}_{\mathrm{ind}} = -\mathrm{j}\omega\gamma(T)\underline{\boldsymbol{A}}$.

In (1), $\underline{\boldsymbol{A}}$ is the phasor of magnetic vector potential $\boldsymbol{A}$ and $\gamma(T)$ is the temperature-dependent electric conductivity; in (2), T is the temperature, $\lambda(T)$ is the thermal conductivity, $\rho(T)$ is the mass density, $c_{\mathrm{p}}(T)$ is the specific heat at a constant pressure, and p_{J} is the volume Joule loss produced in the charge. In Fig. 2 the thermal conductivity-temperature curve of the charge is shown. Apparently, Eq. (1) is solved in the frequency domain while Eq. (2) is solved in the time domain, because the electromagnetic time constant is much smaller than the thermal one. The temperature-dependent physical parameters should be updated at each time step of the computation process [5].

Eq. (1) is subject to Dirichlet condition $\boldsymbol{A} = \mathbf{0}$ along all the boundaries of the magnetic domain (axis of the arrangement and artificial boundary of the air region). In turn, Eq. (2) is subject to conditions: $\frac{\partial T}{\partial n} = 0$ along the axis of symmetry ($r = 0$), and $-\lambda(T)\frac{\partial T}{\partial n} = h(T - T_{\text{ext}})$ along the remaining boundaries of the thermal domain (i.e., the ceramic crucible). The convective parameter h is assumed to be constant and equal to 5 $\text{Wm}^{-2}\text{K}^{-1}$. Moreover, since the charge is inside a ceramic crucible, whose thermal conductivity is very poor, radiation can be neglected.

The initial temperature of the charge is set to $T_0 = T_{\text{ext}}$ (temperature of ambient air).

Two approximations to the aforementioned analysis problem have been considered, namely:

- all the material properties (electrical and thermal conductivities, specific heat and mass density) are constant;
- the electrical conductivity is constant, while the thermophysical properties are temperature dependent (coupled non-linear problem).

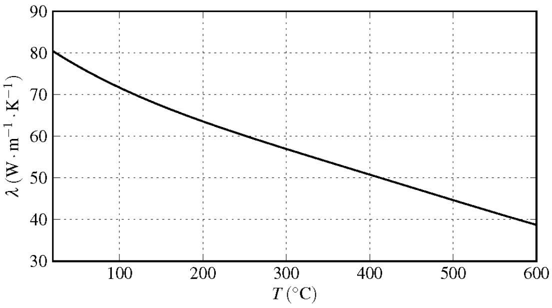

Fig. 2. Thermal conductivity of the charge *vs.* temperature

III. Multiobjective Inverse Problem

The optimal design problem is formulated as follows: having defined all the other parameters in Fig. 1, for a prescribed charge volume find the family of vectors $x = (r, a_1, a_2, f)$ being $f = \frac{\omega}{2\pi}$, such that the following objective functions are optimized in the Pareto sense. Specifically, the normalized heat Q delivered to the charge of volume V in a given time interval $(0, t_{\text{f}})$

$$Q = V^{-1}\rho^{-1}(T_0)c^{-1}(T_0)T_0^{-1}\int_V \left[\rho(T)c(T)(T - T_0)\right]_{t=t_{\text{f}}} \mathrm{d}V \quad (3)$$

must be maximized. Moreover, the temperature field in the charge at the end of the heating process (*i.e.*, at time t_{f}) must be as uniform as possible or, equivalently, the normalized discrepancy

$$U = V^{-1}T_0^{-2}\int_V \left[(T - T_{\text{m}})^2\right]_{t=t_{\text{f}}} \mathrm{d}V\,, \quad (4)$$

is to be minimized, where T_{m} is the time-dependent mean temperature of the charge. In (3)–(4), the temperature distribution T, the mean temperature T_m and the volume V depend on design vector x, i.e. $T = T(t, x_1, x_2, r, a_1, a_2, f)$, $T_m = T_m(t, r, a_1, a_2, f)$ and $V = V(r, a_1, a_2)$; therefore, Q and U are objective functions dependent on design vector x through T, T_m and V. Finally, in Eqs. (3) and (4), T_0 is equal to 20 °C, t_{f} is equal to 30 minutes. The multi-objective optimization problem was solved by means of the NSGA-II algorithm [6]. The solution started from 20 feasible individuals while 20 solution points were found in 20 generations for each problem (linear and coupled non-linear one).

IV. Optimization Results

The set of solutions approximating the Pareto front of the two problems (the linear one with ρ, c_{p}, λ and γ constant, and the thermally non-linear one, with ρ, c_{p}, λ dependent on temperature, while γ is constant) is shown in Fig. 3.

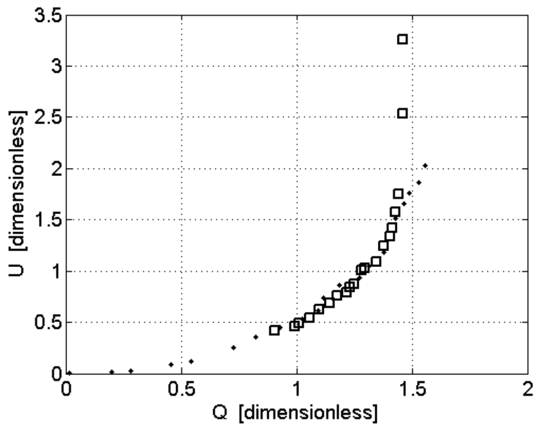

Fig. 3. Optimization results, linear (square), non-linear case (dot)

V. Conclusion

The results obtained after an exploration of the objective space based on the non-linear field analysis are not too far from those based on linear analysis, which is less costly. Therefore, it is reasonable to state that a linear analysis model is sufficient for the purpose of optimal design, while a non-linear analysis can be used *a posteriori*, just for assessing the results of the optimization procedure.

In the full-length paper, the simultaneous dependence of electrical conductivity and thermophysical properties of the heated material on temperature will be considered.

References

[1] P. Di Barba, F. Dughiero, S. Lupi, and A. Savini, "Optimal Shape Design of Devices and Systems for Induction Heating: Methodologies and Applications", COMPEL, vol. 22, no. 1, 2003, pp. 111–122.

[2] P. Di Barba, F. Dughiero, E. Sieni, "Synthesizing Distributions of Magnetic Nanoparticles for Clinical Hyperthermia", *IEEE Trans. Magn.*, 48(2), 263–266, 2012.

[3] A. Candeo, F. Dughiero, "Numerical FEM models for the planning of magnetic induction hyperthermia treatments with nanoparticles", *IEEE Trans. Magn.*, 45, 1654-1657, 2009.

[4] P. Di Barba, I. Dolezel, P. Karban, F. Mach, M.E. Mognaschi, A. Savini, "Multiphysics Field Analysis and Multiobjective Design Optimization: a Benchmark Problem", Proc. OIPE-2012, September 19-21, 2012, Ghent (Belgium), pp.106-107.

[5] J. P. Holman. Heat Transfer. McGraw Hill, NY, 2002.

[6] K. Deb. Multi-Objective Optimization using Evolutionary Algorithms. Wiley, 2001.