An automated robust optimization approach based on robust constraints and objective function

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***Abstract* — This paper introduces a new robust optimization approach for product design. It allows to take into account the variations on both the objective function and the constraints issued from the specifications. It offers the designer a support to choose a compromise solution that fits his requirements in terms of robustness and performance. The design models covered by the proposed approach may be black box (finite element for example) or white box (analytical equations for example).**

***Index Terms* — Uncertainty, Robustness, Design Optimization**

I. Introduction

Despite the efforts provided to master the production processes, industrial products still face uncertainties related to the geometric parameters and materials properties. Ignoring these uncertainties during the design phase can lead to defective products and additional costs.

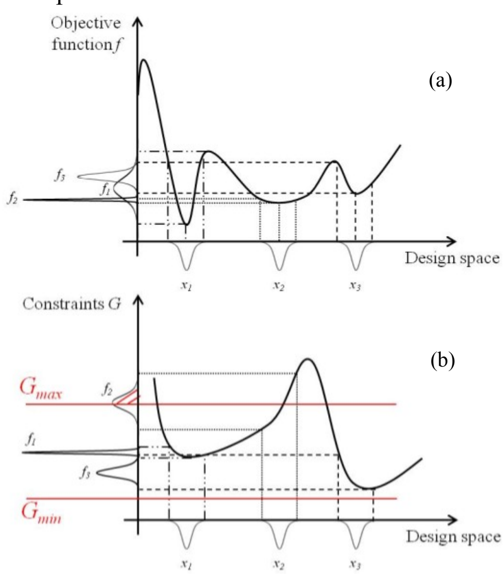

Fig. 1. Robustness in the objective function (a) and constraints (b)

Robust optimization aims to find the best product design taking these uncertainties into account. Usual industrial practices consist in a classical optimization followed by a sensitivity analysis. The global optimum x_1 (Fig. 1.a) is not necessarily robust (too large performance variations f_1 induced by small variations of x). Usually, some design parameters are slightly moved in order to improve the solution sensitivity. Naturally, this allows to explore solutions only near the global optimum which leads, in many cases, to avoid the robust optimal solution x_2. The robust solution corresponding to the local optimum x_2 leads to tight performance variations f_2.

The constraint robustness, rarely addressed in the robust design field, is as important as the robustness on the objective function. Even if x_2 is robust regarding the objective function, it is not robust relating to the constraints G (see Fig. 1.b). This leads x_2 to violate the constraint ($> G_{max}$). The solution x_3 represents a good alternative as it is robust regarding both the objective function f and the constraint G.

The main contribution of this paper is a new robust optimization approach to avoid these drawbacks. The uncertainties propagation is integrated within the optimization loop to directly target the robust optimum satisfying robust constraints. This allows a better exploration of the solutions space. Its automation makes the solution less influenced by the designer and leads to a fast calculation of the robust optimal design. The models covered by the proposed approach may be black box or white box.

II. Robust Optimization approach

A. The flowchart description

The Robust Optimization approach is depicted in Fig. 2. If the design model is computationally costly (3D FEM for example), it can be replaced by a meta-model less time consuming. The Kriging [1] is a response surface method available among others.

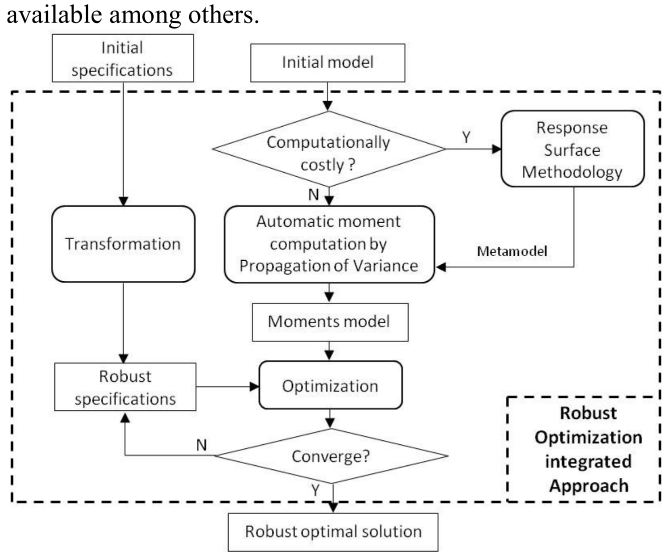

Fig. 2 The Flowchart algorithm

For a fast computation, the variation of the parameters X are characterized by the two first statistical Moments; the Means μ_X and Standard Deviation σ_X. The statistical Moments of the output parameters Y are computed from the input ones using the Moment model $(\mu_Y, \sigma_Y) = M(\mu_X, \sigma_X)$. It is obtained by automatically transforming the initial model using the Propagation of Variance method (PoV) [2].

The Initial specifications are also transformed to handle both the constraints and the objective function robustness.

The optimization considers both the Moment model and the Robust specifications. Depending on the model type (white box or not), one can use either deterministic [3] or stochastic [4] optimization algorithms. In the example presented further, an adapted Particle Swarm Optimization (PSO) algorithm is proposed as it doesn't need the derivative information [4]. The particularity of our PSO lies in its constraint-handling mechanism that consists of a closeness evaluation of each particle to the feasible region. This mechanism "pushes" the particles towards the feasible region using interval arithmetic and then focuses on the objective function to improve the particles' fitness while fulfilling the constraints.

B. *Robust model and specifications*

Recent works in robust optimization propose various robust formulations [5]: Worst-Case formulation, Gradient Index Optimization... In this work, the Initial model and specifications are transformed into the Robust model and specifications as follows:

Initial model and specifications	Robust model and specifications
$\begin{cases} Minimize\ \ f(X) \\ Subject\ to: Y = G(X) \\ Y_L \leq Y \leq Y_U \\ X_L \leq X \leq X_U \end{cases}$	$\begin{cases} Minimize\ \ f_r(\mu_X, \sigma_X) \\ Subject\ to: (\mu_Y, \sigma_Y) = G_r(\mu_X, \sigma_X) \\ [\mu_Y - 3\sigma_Y; \mu_Y + 3\sigma_Y] \subseteq [Y_L; Y_U] \\ [\mu_X - 3\sigma_X; \mu_X + 3\sigma_X] \subseteq [X_L; X_U] \end{cases}$

where f is the objective function, X are the input parameters, Y are the output parameters, G are the constraints, Y_L and Y_U are the lower and upper bounds of Y, and X_L and X_U are the lower and upper bounds of X.

The robust objective function f_r is [6] :

$$f_r(\mu_X, \sigma_X) = \alpha.\frac{\mu_f}{\mu_f{}^*} + (1 - \alpha).\frac{\sigma_f}{\sigma_f{}^*} \qquad (1)$$

where $\mu_f{}^*$ and $\sigma_f{}^*$ are obtained by minimizing respectively μ_f and σ_f. The factor α $(0 \leq \alpha \leq 1)$ expresses a specific tradeoff between the performance and the robustness of f.

The robust constraints G_r, obtained by the Propagation of Variance method, evaluate output statistical Moments (μ_Y and σ_Y) from input ones, as in (2). It is expressed from a second order Taylor expansion [2].

$$G_r(\mu_X, \sigma_X): \begin{cases} \mu_Y \approx G(\mu_X) + \frac{1}{2}\sum_{i=1}^{n} \frac{\partial^2 G}{\partial x_i^2}(\mu_X)\sigma_{x_i}{}^2 \\ \sigma_Y{}^2 \approx \sum_{i=1}^{n} \left(\frac{\partial G}{\partial x_i}(\mu_X)\right)^2 \sigma_{x_i}{}^2 + \frac{1}{2}\sum_{i=1}^{n} \left(\frac{\partial^2 G}{\partial x_i^2}(\mu_X)\right)^2 \sigma_{x_i}{}^4 \\ \qquad + \sum_{i<j}^{n} \left(\frac{\partial^2 G}{\partial x_i \partial x_j}(\mu_X)\right)^2 \sigma_{x_i}{}^2 \sigma_{x_j}{}^2 \end{cases} \qquad (2)$$

where n is the model dimension.

With white box model, exact derivatives are available which fosters the use of deterministic algorithms [3]. With black box, the Finite Difference method performs the first and second derivatives by computing $(2n^2 + 3n)$ the initial model. Consequently, the PoV method is less evaluation consuming compared to the traditional Monte-Carlo simulation (10^6 trials). This is all the more important that it is coupled with optimization algorithm.

III. NUMERICAL RESULTS AND DISCUSSION

Numerical tests have been performed on the robust optimal design of a permanent magnet motor [2]. We consider the analytical model with 10 degrees of freedom. The Initial specifications consist in minimizing the volume Vu while satisfying the parameter domains.

The Robust optimization approach has been implemented in Matlab coupled with the industrialized optimization software Pro@DESIGN [7]. The PSO algorithm was run with 4 particles and 50 iterations. 20 runs were performed for each test which corresponds to 4000 model evaluations.

In order to emphasize the interest of our approach, a classical procedure is applied and compared with our proposal. The classical procedure consists in a non-robust optimization followed by a sensitivity analysis.

The classical optimization gives a non-robust optimal solution. The results of the sensitivity analysis by Monte-Carlo (10^6 trials) are $\mu_{Vu} = 6.07\ e^{-4}$ and $\sigma_{Vu} = 8.23\ e^{-6}$ (see Fig. 3). To improve the robustness of this solution (decrease σ_{Vu}, see Fig. 1.a), numerous analysis have to be performed in order to identify on which design parameters and directions to play.

With our approach, the tradeoff between μ_{Vu} and σ_{Vu} may be fixed by the designer by setting the value of α in the robust objective function (1). As the choice of α is not obvious, it is also possible to draw the Pareto front which gives the designer more flexibility to choose a solution that fits at best his needs in term of robustness (see Fig. 3).

As illustrated in Fig. 3, the solution with a mean of $\mu_{Vu} = 6.12\ e^{-4}$ and a standard deviation of $\sigma_{Vu} = 7.60\ e^{-6}$ can be a good compromise. This compromise solution leads to a loss of 0.8% in the mean and a gain of 7.6% in the standard deviation compared to the non-robust solution. This solution allows a better control of the volume variability while being near the optimal mean (0.8%). In the full paper, more results and in-depth discussion concerning the potential loss of the performance and gain of the robustness will be provided.

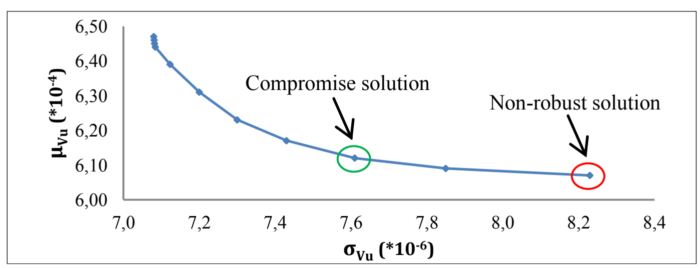

Fig. 3 The Pareto front: μ_{Vu}vs σ_{Vu}

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