Composite First-Order Reliability Method for Efficient Reliability-Based Optimization of Electromagnetic Design Problems

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***Abstract*—This paper proposes a composite first-order reliability analysis method to effectively perform the reliability-based optimization of electromagnetic design problems. The proposed method utilizes both of two different ways, reliability index approach and performance measure approach, for the reliability analysis of probabilistic constraints. The first approach checks the status of probabilistic constrains and the second evaluates the reliability of only active constraints selected from the feasibility identification. That can substantially enhance computational efficiency during optimization process. The proposed method is tested with the TEAM Workshop Problem 22 and its efficiency is compared with existing methods.**

***Index Terms*— Electromagnetics, optimization, reliability theory.**

I. Introduction

The existence of uncertainty in electromagnetic (EM) devices due to manufacturing processes and operational conditions causes subsequent variances in product performances. To systematically incorporate such the uncertainty into an early design stage, a probabilistic design method called reliability-based design optimization (RBDO) has been developed in other engineering fields, such as mechanics and aerodynamics, for the last decade. Usually, the RBDO formulation involves an objective function as deterministic optimization and also contains probabilistic performance constraints for considering the probability of the satisfaction /failure of output performances. In literature [1]-[3], as the first-order reliability method, the reliability index approach (RIA) or the performance measure approach (PMA) has been widely used to deal with the probabilistic designs. It has been reported that RIA often yields instability but PMA is robust in identifying a probability failure mode in the RBDO process. Moreover, both of the two methods require a significant computational cost due to the reliability analysis.

To overcome the aforementioned defects, a composite reliability method, consisting of both RIA and PMA, is newly proposed for the effective RBDO. From the numerical efficiency and accuracy point of view, the proposed method is compared with existing RBDO methods adopting either RIA or PMA.

II. Composite Reliability method for RBDO

The proposed method utilizes both of the two different reliability analysis methods to substantially improve the computational efficiency of the reliability analysis in the RBDO process. A different role is assigned to each method: while RIA just checks the feasibility status of the constraints, PMA mainly evaluates the reliability of only active constraints selected from the feasibility identification of RIA.

A. Feasibility identification

In the system parameter design, the RBDO model can be generally defined by [1] and [2]

$$\begin{aligned} &\text{minimize} && f(\mathbf{d}) \\ &\text{subject to} && P(g_i(\mathbf{x})>0)\le P_{t,i}, \quad i=1,2,\ldots np \qquad (1) \\ & && \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \mathbf{d}\in R^n \end{aligned}$$

where f is the objective function, $\mathbf{d}$ is the design variable vector given by $\mathbf{d}=\mu(\mathbf{x})$, μ denotes the mean value vector of the random vector $\mathbf{x}$, and $P_{t,i}$ is the target failure probability with respect to the ith constraint function g_i. The symbols, $\mathbf{d}^L$ and $\mathbf{d}^U$, mean the lower and upper bound of $\mathbf{d}$, respectively. Unlike deterministic optimization, RBDO must take into account the feasibility of probabilistic constraints at a design point through the reliability analysis. That mainly causes a considerable computational burden in the RBDO process.

For the purpose of eliminating unnecessary reliability analysis, this paper exploits RIA as a feasibility identifier for probabilistic constraints. The feasibility check scheme for probabilistic constraints is illustrated in Fig. 1 where β_t is the target reliability index with respect to the constraint g in the standard normal design space (U^0-space) of an initial design.

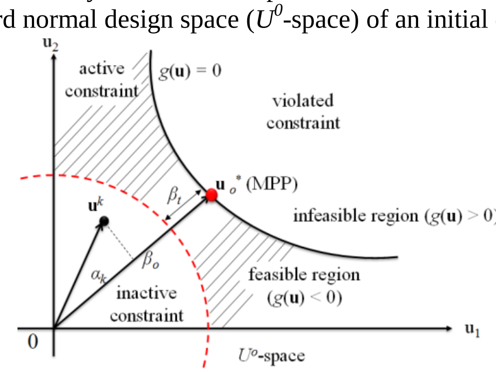

Fig. 1. Identifying feasibility status using RIA.

A most probable failure point (MPP), $\mathbf{u}_o^*$, and the reliability index β_o are obtained at the initial design after carrying out RIA. The improved kth design point $\mathbf{d}^k$ is transformed into $\mathbf{u}^k$ in the U^0-space and then α_k is defined by the inner product of two distance vectors, $\mathbf{u}_o^*$ and $\mathbf{u}^k$, without the reliability analysis. From the spatial information between design points in the U^0-space, the feasibility status of probabilistic constraints at each iterative design is defined as:

1) Inactive probabilistic constraint if β_o-$\alpha_k > \beta_t$
2) Active probabilistic constraint if $0<\beta_o$-$\alpha_k \le \beta_t$
3) Violated probabilistic constraint if β_o-$\alpha_k > \beta_t$

In the proposed method, the reliability analysis is carried out only for active probabilistic constraints.

B. Program architecture

The implementation of the proposed RBDO method consists of a double-loop optimization structure as shown in Fig. 2. The optimization problem of (1) has a sub-optimization problem for the reliability analysis of probabilistic constraints at each iterative design. Therefore, the procedure of the RBDO problem is divided into two optimization loops as:

1) Inner loop: sub-optimization for evaluating the failure probability of each constraint (dotted box in Fig. 2),

2) Outer loop: overall optimization to optimize the objective function with probabilistic constraints.

The distinctive of the method is that, at each iterative design, RIA just checks the feasibility identification and then PMA mainly evaluates the constraint reliability.

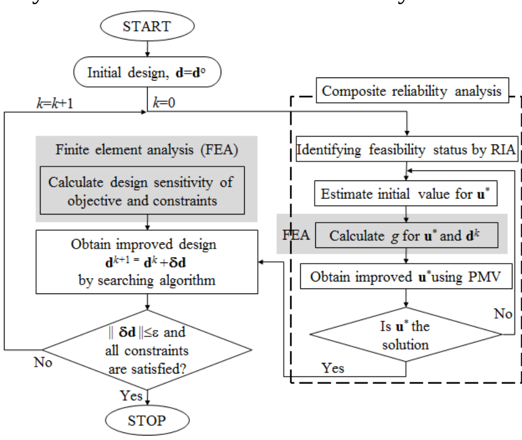

Fig. 2. Flowchart of the proposed RBDO method.

III. Results

To examine the efficiency of the proposed method, the TEAM benchmark problem 22 in Fig. 3 is considered [4]. For simplification of the design problem, only three of total eight design variables, R2, D2 and H2, are selected as independent random variables. The RBDO formulation is written by:

$$
\begin{aligned}
&\text{minimize } f(\mathbf{d}) = \sum_{i=1}^{21} |B_{stray,i}(\mathbf{d})|^2 \\
&\text{subject to } P(g_i(\mathbf{X})>0) - \Phi(-\beta_{t,i}) \le 0 \quad i=1,2,3,4 \\
&g_1(\mathbf{X}) = 1 - \left(\frac{E(\mathbf{X}) - E_o}{0.05 \times E_o}\right)^2, \; g_2(\mathbf{X}) = (R_2 - R_1) - \frac{1}{2}(D_2 + D_1) \\
&g_{3,4}(\mathbf{X}) = -|\mathrm{J}_k| - 6.4|\mathrm{B}_{\max,k}| + 54.0 \quad k=1,2
\end{aligned}
\tag{2}
$$

where $B_{stray,i}$ is the stray field calculated at the ith measurement point along line a and line b, E is the stored magnetic energy with a target value E_o of 180 MJ, and the wanted confidence level $\beta_{t,i}$ is set to be 1.645 corresponding to the failure probability value of 5% (i.e. reliability of 95%). It is assumed that the random variables follow the normal distributions and the SD values of R_2, D_2 and H_2 are 10mm, 5mm and 10mm, respectively, as presented in Table I.

The optimization problem was solved using three different RBDO methods which are based on RIA, PMA and proposed composite method, respectively. The design sensitivity values for the RBDO process were calculated with the finite differencing method where a commercial EM simulator, called MagNet VII [5], and an embedded Matlab function were utilized. To deal with the constraint conditions of (2), the sequential quadratic programming algorithm was used. Starting with the same initial design, the obtained optima are presented in Table I. It is observed that the design points and their performance indicators between the PMA-based and the proposed RBDO methods are almost same with each other. Meanwhile, the function calls (i.e. total number of finite element analyses) for convergence are compared in Table II. It is obvious that the proposed method requires the smallest function calls without degrading the accuracy of solutions. It implies that the method effectively eliminates unnecessary reliability analysis parts in the RBDO process.

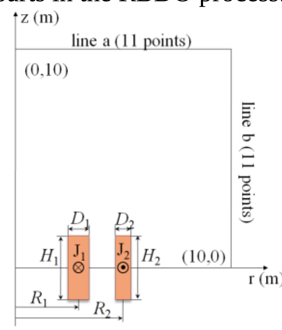

Fig. 3. Configuration of the SMES device.

TABLE I
PERFORMANCE INDICATORS AT FOUR DIFFERENT DESIGNS

Design variables	d^L	SD	d^U	Initial	RBDO RIA	PMA	Proposed
R_2 (mm)	2300	10	2400	2335	2348	2346	2347
D_2 (mm)	200	5	350	238	233	231	232
H_2 (mm)	1600	10	1900	1853	1867	1887	1884
B_{stray} (μT)	-	-	-	32	32	37	37
E (MJ)	-	-	-	173	181	181	181
$P_f(g_1)$	-	-	-	30.80	4.33	4.73	5.16
$P_f(g_2)$	-	-	-	1.66×10^{-2}	1.47×10^{-5}	0	0
$P_f(g_3)$	-	-	-	0	0	0	0
$P_f(g_4)$	-	-	-	0	0	0	0

* The results were obtained with the values: R_1=1977 mm, D_1=404 mm, H_1=1507 mm, J_1=16.30 A/mm^2, and J_2=16.19 A/mm^2.

TABLE II
FUNCTION CALLS BETWEEN THREE DIFFERENT RBDO METHODS

Constraint	RBDO RIA	PMA	Proposed
$P_f(g_1)$	1853	696	492
$P_f(g_2)$	585	464	178
$P_f(g_3)$	1017	676	121
$P_f(g_4)$	1097	676	121
Function calls	4552	2552	912

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