Topology Optimization Using Material Density Based on Sigmoid Function by Means of Sequential Linear Programming

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***Abstract*—Topology optimization is typically solved by material density-based approaches, in which the material density is formulated as a solid isotropic material with penalization. To suppress the generation of intermediate-density material, the penalization factor is set to a value greater than 1. However, the convergence characteristics of objective function deteriorate, and it becomes difficult to derive a high-performance topology. Thus, we propose that the material density can be represented by a sigmoid function. This paper demonstrates the effectiveness of the sigmoid function in suppressing the generation of gray scale.**

***Index Terms*—design optimization, finite element methods, linear programming, magnetostatic shielding, topology**

I. Introduction

Topology optimization (TO) has the useful ability to realize a more flexible shape of magnetic circuit than can be derived from size or shape optimization. Furthermore, TO permits the discovery of novel geometries, because the optimized solution is not dependent on conventional information.

The material density approach [1], in which the binary information for material allocation is transformed into the continuous space, has been widely applied to electromagnetic systems [2] – [4]. Generally, the material density is formulated as a solid isotropic material with penalization (SIMP) [5], which can suppress the generation of intermediate-density material (gray scale) by increasing the penalization factor. However, this increment degrades the performance of the converged topology [5], [6].

This paper proposes a density-based TO in which the material density is formulated according to a sigmoid function to suppress the generation of gray scale. The effectiveness of the proposed method is demonstrated by a comparison with a density-based SIMP TO in a 2-D magnetostatic shielding problem.

II. Density-based Topology Optimization

In material density-based TO, the material density of a finite element is adopted as the design variable. When the SIMP is applied to a material of density ρ_i in an element i, the linear permeability μ_i is given as follows:

$$\mu_i = \mu_0\{1+(\mu_r-1)\rho_i^{\,n}\}\,, \tag{1}$$

where μ_0 is the permeability in a vacuum, μ_r is the relative permeability of the magnetic material, and n is a penalization factor to suppress the generation of gray scale elements.

Similarly, the permeability using a density-based sigmoid function $\varsigma(\rho_i)$ can be formulated as

$$\mu_i = \mu_0\{1+(\mu_r-1)\varsigma(\rho_i)\}\,. \tag{2}$$

Then, $\varsigma(\rho_i)$ is defined as follows:

$$\varsigma(\rho_i) = \frac{1}{1+e^{\{-\alpha(\rho_i-0.5)\}}}\,, \tag{3}$$

where α is a factor for controlling $d\varsigma(\rho_i)/d\rho_i$ at $\rho_i = 0.5$. We set α to 15 in order to clarify the partition between μ_0 and $\mu_0\mu_r$ clear.

Fig. 1 shows the change in permeability and the derivative with respect to ρ. The sigmoid behavior of μ_i has a point symmetry arrangement about $\rho_i = 0.5$, and the magnitude of $d\mu_i / d\rho_i$ attains its maximum value at $\rho_i = 0.5$. Because the permeability of gray scale elements at $\rho_i = 0.5$ has the mobility to become μ_0 or $\mu_0\mu_r$, it is expected that the gray scale elements can be eliminated. Then, assuming that the material density is constantly distributed in a finite element, the TO can be performed.

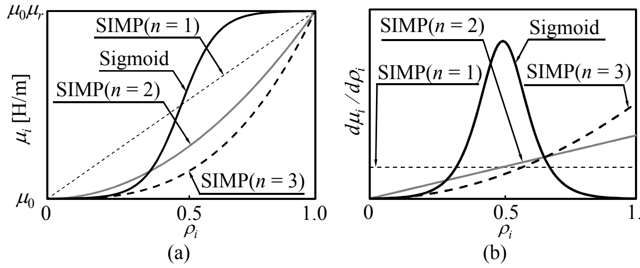

Fig. 1. Changes of permeability and its derivative: (a) permeability, (b) the derivative of permeability with respect to ρ_i.

III. Optimization Model and Design Goal

A. 2-D Box Shield Model

We adopt a box shield model as the 2-D optimization target in the magnetostatic field, as shown in Fig. 2. The analyzed region is reduced to a quarter area using the shape symmetry. Analysis scale reduction is performed using a nonconforming mesh connection with a linear combination. The allocated magnetic material is assumed to have linear properties, and μ_r is set to 10^3. Then, the design domain is composed of 6,912 square elements, and the analyzed region is discretized by 10,356 quadrilateral elements.

B. Objective Function

The TO goal is to minimize the magnetic energy in the target domain, under the condition that the area S of the magnetostaic shielding is less than the specified area S_0. The formulation of objective function W can be defined as follows:

$$\begin{aligned} &min. && W(\boldsymbol{A},\rho)=\frac{1}{2}\iint_{S_t}\boldsymbol{B}^2/\mu_0\,dS \\ &s.t. && 0\le\rho_i\le 1\quad(i=1,\cdots,n_d)\quad, \\ &&& S(\rho)=\iint_{S_d}\rho\,dS\le S_0 \end{aligned} \tag{4}$$

where S_t is the area of the target domain, $\boldsymbol{B}$ is the magnetic flux density, n_d is the number of design variables, and S_d is the area of the design domain. Next, (4) is transformed into a linear problem using a Taylor expansion with respect to $\delta\rho^{(k)}$ which is the correction to $\rho^{(k)}$ in the k-th iteration, as follows:

$$\begin{aligned} &min. && \nabla W(\boldsymbol{A}^{(k)},\rho^{(k)})^T\delta\rho^{(k)} \\ &s.t. && \max[-\rho_i,-\zeta^{(k)}]\le\delta\rho^{(k)}\le\min[1-\rho_i,\zeta^{(k)}] \\ &&& \qquad\qquad\qquad\qquad\qquad (i=1,\cdots,n_d)\;, \\ &&& S(\rho^{(k)})+\nabla S(\rho^{(k)})^T\delta\rho^{(k)}\le S_0 \end{aligned} \tag{5}$$

where ∇ is the Hamiltonian operator with respect to $\rho^{(k)}$, and $\zeta^{(k)}$ is a move limit for $\rho^{(k)}$. The linear problem can be solved by sequential linear programming. When $\delta\rho^{(k)}$ is less than 10^{-2}, the TO is stopped.

IV. Optimization Results

The TO of magnetostaic shielding is performed under the condition that S_0 is set to the value of one-third the design domain with $\rho^{(0)}$ set to 0.33. The judgment for gray scale is based on the permeability range $0.2\mu_0\mu_r < \mu_i < 0.8\mu_0\mu_r$.

The optimized topologies are shown in Fig. 3. It can be seen that the magnetic shielding is comprehensively composed of a multi-layered construction. Whereas many gray scale elements appear in the topology of SIMP (n = 1), there are few gray scale element in the results from SIMP (n = 2) or SIMP (n = 3). The penalization factor is useful for the suppression of gray scale generation. Furthermore, the reduction of the gray scale is confirmed for sigmoid function. As shown in Fig. 4, the sigmoid function has similar performance to that of SIMP at n = 2, 3. The reason why the iron constitution (= 0.41) at Sigmoid case is larger than 1/3 is that the higher value of μ_i is not comparable to the material density as shown in Fig. 1.

Fig. 5 shows the convergence characteristics of W. When the n value of SIMP increases, the convergence history deteriorates. However, the characteristics of the sigmoid function are better than the results from SIMP with n = 2, 3.

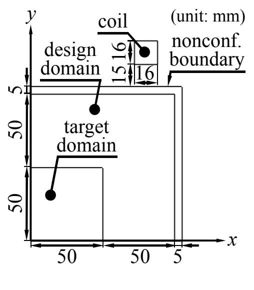

Fig. 2. Optimization model.

The converged value of W has similar properties, as shown in Table I. It is likely that there is no disadvantage in using a material density definition based on the sigmoid function. The performance of a large scale 3-D problem will be addressed in the full paper.

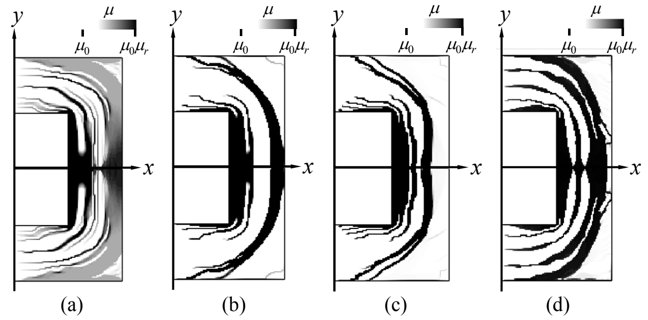

Fig. 3. Optimization topologies: (a) SIMP (n = 1), (b) SIMP (n = 2), (c) SIMP (n = 3), (d) Sigmoid.

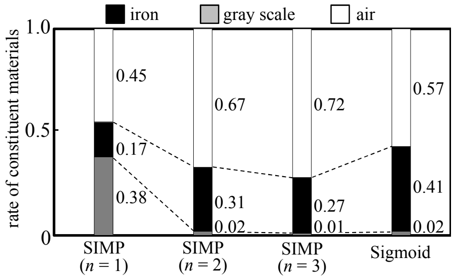

Fig. 4. Rates of constituent materials in optimized topologies.

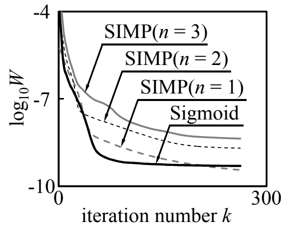

Fig. 5. Convergence characteristics of the objective function.

TABLE I
Optimization Results

	$\rho^{(0)}$	ite.	elapsed time [s]	$W\times 10^{-10}$ [J]	$S(\rho)/S_0$
SIMP(n = 1)	0.33	262	85.1	3.621 (0.17)	1.00
SIMP(n = 2)		262	86.2	20.23 (1.00)	1.00
SIMP(n = 3)		262	86.4	43.20 (2.13)	0.99
Sigmoid		262	87.0	4.971 (0.25)	1.00

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